Drift Analysis of Backlogged Packets in Slotted ALOHA

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***Abstract*—These Multiple access system (MAS) deals with a situation where multiple nodes (computers) are required to access commonly shared channel. These multiple users can be viewed as uncoordinated users or computers which compete to transmit messages in the form of packets or frames. It can be shown that the MAS is unstable due to nonlinearity of the problem in the form of contention of several nodes competing for the same channel. There are some well known algorithm to stabilize the throughput of slotted ALOHA i.e bayesian broadcast algorithm, splitting algorithm, modified stochastic gradient algorithm. In this paper, we have analyzed the number of backlogged packets by using statistical approach and stabilize the expected number of backlogged packets minus mean number of packets which are successfully transmitted.**

***Index Terms*—Slotted ALOHA, throughput, backlogged packets, probabilistic distribution, diffusion approximation.**

I. Introduction

ALOHA protocol is simple communications scheme where each source (transmitter) in a network sends data in the form of frame or packet. If a packet successfully reaches the destination (receiver), the next packet is sent. If the frame or packet fails to be received at the destination, it must have suffered collision and becomes backlogged and is to be retransmitted. Both fresh packets and backlogged packets are transmitted and retransmitted respectively with certain probability distribution. ALOHA protocol is a generic protocol which is inherently unstable and is referred to as random access communication system. The stabilization of the slotted ALOHA (S-ALOHA) protocol is an important problem. Several stabilization methods have been proposed such as splitting algorithm, Bayesian broadcast method and modified stochastic gradient algorithm [1-3]. Analysis of the stability of the finite population model of the slotted ALOHA scheme shows that this system can only have: (1) one stable equilibrium point or (2) three equilibrium points with the first one and the third one being stable and the middle one unstable [4]. An interesting result shows that the system has only one stable equilibrium point provided the transmission probability is less than or equal to 2/N, where N is the total number of active users in the system. It has been established that the input/output packet flow balance principle and the concept of expected drift used for the stability analysis are mathematically equivalent [5-7]. In the performance analysis of ALOHA and related schemes, the analysis is based on the assumption that buffer capacity is of one unit [8]. This results in the state being either idle or busy. The problem becomes unwieldy when the buffer size is finite and leading to a very large number of interdependent of states (Sykas, Karvelas and Protonotarios [9]). This makes the analysis of finite buffer systems intractable. The ALOHA and slotted-ALOHA protocols are known to be unstable. This aspect of instability has been extensively discussed in the literature **[**4**]**. For the sake of completeness, we briefly discuss the analytical framework. In a slotted ALOHA, number of backlogged packets can be considered as a random variant. Now, consider this random parameter called as number of backlogged packets is following some probabilistic distribution. It can be easily transformed on to range [0, 1]. Several procedures have been suggested to stabilize slotted ALOHA. We briefly describe some of the well known algorithms for this purpose [2]. In this paper, we have shown that this mapped value is following uniform and normal distribution.

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II. Model Description

A. Transmission Probability for Slotted Aloha

We are analyzing the result that whether expected change in number of backlogged packets are affected or not. We consider a ALOHA channel with m users. As discussed earlier, a node will transmit fresh packet with probability q_a while a backlogged packet (which has suffered collision) with probability q_r. The number of backlogged users at time length t is denoted by stochastic process {n(t), t € T}. The probabilistic description of {n(t)} is governed by Markov Chain. Following [10-11], The one step transition probability matrix for the stochastic process n(t).

$$P_{i,j} = P[n(t+1) = j \mid n(t) = i] \tag{1}$$

This is a finite Markov chain with nonlinear transition rates which make the problem really difficult to analyze. In order to make the problem tractable, approach based on diffusion approximation has been employed see [10-11].

The average throughput per slot would be

$$\begin{aligned} S(n,p,N) &= p_{n,n-1} + (N-n)p(1-p)^{N-1} \\ &= Np(1-p)^{N-1} \end{aligned} \tag{2}$$

Jenq[3] has been carried out detailed analysis and found that for large N, maximum throughput occurs when p = 1/N. This reduces to the form which is similar to slotted ALOHA system viz.

$$S_{max}(n,p,N) = (1-\tfrac{1}{N})^{N-1} = \tfrac{1}{e} \tag{3}$$

But the requirement is that N should be known to the controller. A new algorithm for stabilization of slotted ALOHA employs modified stochastic gradient algorithm (MSG). This algorithm updated the parameter characterizing the permission probability p during each time slot. Reyes, McLernon and O'Farrell [2] have proposed a variant of stochastic gradient algorithm.

Denoting by p_k as the permission probability during k^{th} slot, the update approach is based on the following update scheme.

$$p_{k+1} = p_k - \mu \frac{\delta J_k}{\delta p_k} \quad (4)$$

where μ is step size and J_K is the criterion function.

This criterion function will be responsible to minimize the error and indirectly indicate the expected number of packet lost.

B. Diffusion Approximation Approach

As said, it is analytically convenient to approximate discrete Markov Chain of backlogged packets by continuous time process. Such an approximation is called diffusion approximation which is governed by a stochastic differential equation. Kaj[7] has discussed diffusion approximation approach in the context of slotted ALOHA and CSMA systems. For the sake of completeness we describe in detail the approach for slotted ALOHA system.

The diffusion approximation approach requires the system size to become infinitely large. The size of the system is characterized by the number of user m. In the limiting case when m→∞, one scales the probabilities of transmission and retransmission as

$$q_a = \lambda/m\ ,\ q_r = \alpha/m \quad (5)$$

Assuming λ, α to be O(1), then q_a and q_r are O(1/m).
The proportion of backlogged packets $\{ x_t^{(m)} \}$ in k^{th} slot is obtained from the ratio $N_k(t)$ / m, where, $N_k(t)$ denotes the number of backlogged packets in k^{th} slot.

It may be noted that N_k has used for the backlogged user in n^{th} slot. Denoting the number of backlogged users in k^{th} slot by Z_k and the number of new packet arrivals in the channel from free users by A_k, it can be find that in k^{th} slot,

$$N_{k+1} = N_k + A_{k+1} + (Z_{k+1} + A_{k+1} = 1) \quad (6)$$

The random variable system Z_k and A_k conditional on N_k are independent and Binomially distributed such that

$$Z_{k+1} \sim B_{in}(N_k, q_r) \quad (7)$$

$$A_{k+1} \sim B_{in}(m - N_k, q_a) \quad (8)$$

This crucial observation enables us to make further investigations about the process $\{ x_t^{(m)} \}$, where the superscript m denote the size of the system. Scaling time, such that k=[m t], then

$$X_t{}^{(m)} = \frac{1}{m} N_{[mt]} \quad (9)$$

There can be computed the drift and quadratic variation function as

$$D_m(x) = \frac{1}{h} E\left[X_{t+h}{}^{(m)} - X_t{}^{(m)} \middle| X_t{}^{(m)} = x\right] \quad (10)$$

$$\sigma_m{}^2(x) = \frac{1}{h} E\left[\left(X_{t+h}{}^{(m)} - X_t{}^{(m)}\right)^2 \middle| X_t{}^{(m)} = x\right] \quad (11)$$

where h=1/m.

Using equation (7) and (8) , we finally obtains[7].

$$D_m(x) = lim_{m\to\infty} D_m(x) = \boldsymbol{\lambda}(1 - x) - (\boldsymbol{\lambda}(1 - x) + \alpha x)e^{-\boldsymbol{\lambda}(1-x)-\alpha x} \quad (12)$$

$$\sigma_m{}^2(x) = lim_{m\to\infty} \sigma_m{}^2(x) = \boldsymbol{\lambda}^2(1 - x)^2 - (\boldsymbol{\lambda}(1 - x) + \alpha x)e^{-\boldsymbol{\lambda}(1-x)-\alpha x} \quad (13)$$

It can be seen in the limit of m→∞, $x_t^{(m)}\}$ converges to the solution of deterministic ordinary differential equation given by

$$\frac{dx(t)}{dt} \equiv D(x, \boldsymbol{\lambda}, \alpha),\ \ x(t = 0) = x_0 \quad (14)$$

III. Effect of Random Parameters in MAS

We have so far discussed random access system when the parameters of the problem remain fixed. The question of interest is to investigate the effect of randomization of parameters on the stability of random access systems. In view of nonlinearity of the problem, it is impossible to obtain explicit time dependent solution. Accordingly, the best one can do under these circumstances is to be resort to simulation study based on Monte Carlo simulation techniques [11]. The question of interest is to examine the stability issues in random access system. For example it would be worth investigating whether random perturbations can introduce new features in system's behavior. To this end we consider in detail ALOHA system. The same analysis can be easily extended to CSMA/CD system.

Drift Function with Random Transmission Probability Parameters

We first examine the effect of random parameter in drift function given by equation (12). We now consider some special cases

Case-1: λ is normally distributed

We assume the parameter λ to be randomly distributed with pdf g(λ) and drift function is given as below.

$$D(x|\lambda) = \lambda(1 - x) - \{\lambda(1 - x) + \alpha x\}e^{-\lambda(1-x)-\alpha x} \quad (15)$$

where, 0 ≤ x ≤ 1

It is straightforward to see that for different realizations of random variable λ, D(x | λ) is also a random variable. Our interest is to obtain expected value of drift i.e.

$$E[D(x)] = \int_{-\infty}^{+\infty} D(x|\lambda)g(\lambda)d\lambda \quad (16)$$

We assume parameter λ to be normally distributed with mean μ and variance σ^2 such as λ~N(μ, σ^2).

Case-2 : α is normally distributed

We now consider another case where retransmission probability is randomized but λ remains fixed. Now drift function can be given as

$$D(x|\alpha) = \lambda(1 - x) - \{\lambda(1 - x) + \alpha x\}e^{-\lambda(1-x)-\alpha x} \quad (17)$$

We can easily compute

$$E[D(x)] = \int D(x|\alpha)\, \phi(\alpha)d\alpha \quad (18)$$

where Φ(α) is pdf of random variable α. We briefly present simulation results in two cases as given in Fig. 1 and Fig.2.

Case-3: λ and α randomly distributed

We now examine the effects when both λ and α are randomly perturbed. Assuming that joint pdf of random variables λ and α to be ψ(λ, α), the expected drift becomes

$$E[D(x)] = \iint D(x|\lambda,\alpha)\,\psi(\lambda,\alpha)\,d\lambda\,d\alpha \tag{19}$$

and drift function is given by

$$D(x|\lambda,\alpha) = \lambda(1-x) - \{\lambda(1-x) + \alpha x\}e^{-\lambda(1-x)-\alpha x} \tag{20}$$

When λ and α are independent, one can write

$$\psi(\lambda,\alpha) = g(\lambda)\phi(\alpha) \tag{21}$$

IV. Simulations and Results

It is to be noted that the functional form of expected drift with randomized λ gets modified. Accordingly, the randomization of parameters is bound to affect system's behavior. As the parameter λ is a positive quantity, the choice of σ should be such that $\mu - 3\sigma > 0$. For particular set of values μ and σ^2, we simulate sample paths for the drift function. Here $\lambda \sim N(0.4, 0.01)$ and α=5. In order to reduce sampling error, we have generated $k=10^4$ sample paths and sampling error will of $O(k^{-1/2})$ (see Glasserman[11]). The results of simulation study are presented in Fig.1 and it shows us, there are different solutions for different value of λ.

We depict the sample paths for the case when $\alpha \sim N(5, 0.09)$ as shown in Fig.2. Similarly, we can also show the simulation for backlogged packets or un-backlogged packets using uniform distribution. It is found that variability with normally distributed parameter is more pronounced than that for uniformly distributed.

In some special cases, one can obtain explicit expression for mean drift. In this case it has been shown that graph for D(x) is cutting only two times instead of three times by introducing randomness in parameter λ and α as shown in Fig.3. In Fig.3, original graph show the result for λ=3.5 and α=5.This result tells us about stabilization of expected change in backlogged packets.

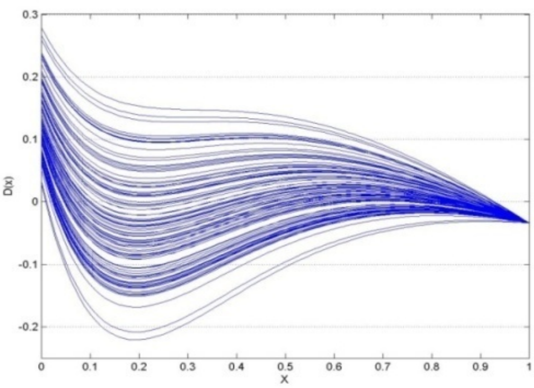

Fig. 1. Variation of D(x) w.r.t. x when $\lambda \sim N(0.4,0.01)$ and α =5

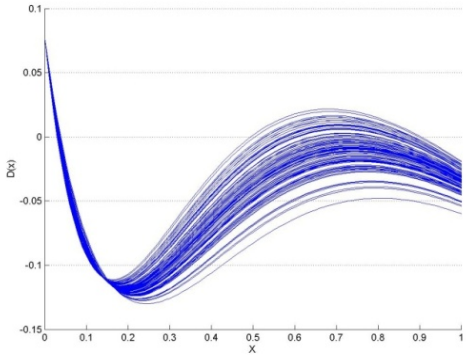

Fig. 2. Variation of D(x) w.r.t. x when $\alpha \sim N(5,0.09)$. and λ=0.3

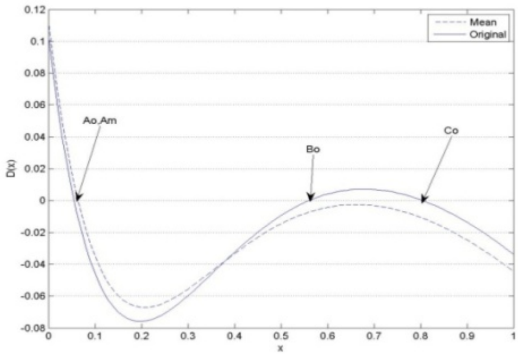

Fig. 3. Stabilization of D(x) when $\lambda \sim U(0.2,0.5)$ and $\alpha \sim U(3.5,6.5)$

V. Conclusions

We have examined the effect of random parameters on the behavior of backlogged packets in random access system, more particularly for slotted ALOHA system. What holds for slotted ALOHA should also hold for CSMA system as both are inherently unstable. We are finding that such random perturbation may significantly affect system's behavior. We find that in particular with both randomized transmission and retransmission probability, system may make a transition from bi-stable (deterministic) to mono-stable (random) behavior. We have found this behavior being exhibited in simulation study. Motivated by this, it would be important to further investigable these cases in analytical setting. As shown by Kaj[7] that for finite m one is required to deal with SDE. In our case we have to simulate SDE with random parameters. It would be useful to carry out extensive simulation study to examine the issue of sensitivity of random access systems under parametric uncertainty. See the Fig.3, the expected values of backlogged packet in slotted ALOHA obtained by Monte Carlo simulation have been changed by introducing randomness in parameter λ and α.

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