Evidence Theory in Incomplete Information Tables

Yu-Ru Syau and En-Bing Lin

Abstract—**In this paper, we study rough set approximations in an incomplete information table via a generalized model of Ziarko's variable precision rough set model, called Variable Precision Generalized Rough Set (VPGRS) model. Viewing the β-lower and β-upper approximations in VPGRS model as mappings from 2^U (the power set of the universe of discourse) to itself, we show that they are mutually dual, and that both of them are order-preserving. We then introduce the belief and plausibility functions, respectively, over *U*, based on the β-lower and β-upper approximations, respectively, in VPGRS model, and we incorporate the concepts of evidence theory and VPGRS model to examine incomplete information tables.**

Index Terms—**Rough sets, belief functions, reflexive relations, variable precision rough set models, lower and upper approximations.**

I. Introduction

Evidence theory is a useful tool in knowledge representation which plays an important role in dealing with many aspects of problem solving. This includes handling incomplete information tables [1]. One of the most important concepts an intelligent system needs to understand is the concept of knowledge. It may or may not be perfect. Also, one wants to know what knowledge is needed to achieve particular goals, and how that knowledge can be obtained. So, one of the important problems along this line is to seek an appropriate approach to analyze imperfect knowledge. The problem related to imperfect knowledge or an incomplete information table has been investigated by many researchers in different areas. Our approach is to apply evidence theory which is essentially Dempster-Shafer theory [2]. This theory is a generalization of the Bayesian theory of subjective probability, also known as the theory of belief functions. Some important features of Dempster-Shafer theory are that it has the capability to cope with varying levels of precision regarding the information and allows for direct representation of uncertainty of system responses where an imperfect information can be characterized by a set or an interval. With these features, we consider the concept of variable precision rough set model [3] that extends applications in lower and upper approximations [4]. Rough set theory [5], [6] can be used to model certain classification of the available information but the classification must be fully correct or certain. We need a method that can handle the classification with some degree of uncertainty. In this paper, we focus on applying belief functions to representing partial knowledge of incomplete information tables [7], [8]. In what follows, we set up the notations and recall lower and upper approximations in Variable Precision of Generalized Rough Sets (VPGRS) [9]-[12]. We then define belief functions and incomplete information tables. We establish several relationships on incomplete information tables by analyzing lower and upper approximations in VPGRS models. We also connect evidence theory and VPGRS models.

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II. Preliminaries

Let U be a nonempty finite set, referred as the universe of discourse (in short, the universe). The power set of U, denoted by 2^U, is the collection of all subsets of U, including the whole set U and the empty set Ø. That is,

$$2^U = \{S \mid S \subseteq U\}.$$

The Cartesian product $U \times U$ is the set of all ordered pairs of elements of U. A binary relation on U is a subset of $U \times U$.

For a relation $R \subseteq U \times U$, we often write xRy to represent $(x, y) \in R$. In case R is an equivalence relation, we say that objects x and y are equivalent.

Let $R \subseteq U \times U$. For each $x \in U$, the image of x under a relation R is defined as $R(x) = \{y \in U \mid xRy\}$. Notice that in case R is an equivalence relation, the images are either disjoint or identical; we use a special notation and write $R(x)$ as $[x]_R$, referred as the R-equivalence class of x. The collection U/R of all distinct R-equivalence classes forms a partition of U, and is referred to as the quotient set of U modulo R.

A. Set Approximations in the VPGRS Model

From now on, unless otherwise specified, we shall assume that $R \subseteq U \times U$ is reflexive. Based on a reflexive relation $R \subseteq U \times U$, Pawlak's lower and upper approximations [5], [6] are commonly extended in the following way [13], [14]:

for $X \subseteq U$,

$$\underline{R}(X) = \{x \in U \mid R(x) \subseteq X\}, \tag{2.1}$$

$$\overline{R}(X) = \{x \in U \mid R(x) \cap X \neq \emptyset\}. \tag{2.2}$$

Let β be a parameter such that $0 \leq \beta < 0.5$. For $x \in U$, and $X \subseteq U$, we define $R(x) \subseteq^{\beta} X$ by

$$R(x) \subseteq^{\beta} X \text{ if and only if } e(R(x), X) \leq \beta$$

which is equivalent to $1 - \frac{|R(x) \cap X|}{|R(x)|} \leq \beta$ or $\frac{|R(x) \cap X|}{|R(x)|} \geq 1 - \beta$,

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where $|\cdot|$ is the set cardinality and

$$e(R(x), X) = 1 - \frac{|R(x) \cap X|}{|R(x)|} \quad (2.3)$$

is the inclusion error of $R(x)$ in X.

For $x \in U$, and $X \subseteq U$, we also define $R(x) \cap^{\beta} X \neq \emptyset$ by $R(x) \cap^{\beta} X \neq \emptyset$ if and only if $e(R(x), U - X) > \beta$ which is equivalent to $1 - \frac{|R(x) \cap (U-X)|}{|R(x)|} > \beta$ or $\frac{|R(x) \cap X|}{|R(x)|} > \beta$.

With the above notations, the β-lower and β-upper approximations in Ziarko's VP-model can be extended in the following way [11].

For $X \subseteq U$,

$$\begin{aligned} \underline{R}^{\beta}(X) &= \{x \in U \mid R(x) \subseteq^{\beta} X\} \\ &= \{x \in U \mid 1 - \frac{|R(x) \cap X|}{|R(x)|} \leq \beta\}, \\ &= \{x \in U \mid \frac{|R(x) \cap X|}{|R(x)|} \geq 1 - \beta\}, \end{aligned} \quad (2.4)$$

and

$$\begin{aligned} \overline{R}^{\beta}(X) &= \{x \in U \mid R(x) \cap^{\beta} X \neq \emptyset\} \\ &= \{x \in U \mid 1 - \frac{|R(x) \cap (U - X)|}{|R(x)|} > \beta\} \\ &= \{x \in U \mid \frac{|R(x) \cap X|}{|R(x)|} > \beta\}. \end{aligned} \quad (2.5)$$

Rough set theory with such approximations will be referred to as the variable precision generalized rough set (VPGRS) model [11]. Notice from (2.1)-(2.5) that for any $X \subseteq U$,

$$\underline{R}^{0}(X) = \underline{R}(X),\ \overline{R}^{0}(X) = \overline{R}(X) \quad (2.6)$$

Using (2.3)-(2.5), we immediately obtain the following relationships for β-lower and β-upper approximations.

Lemma 1. Let $R \subseteq U \times U$ be reflexive, and let $\beta \in [0, 0.5)$. Then

1. $\underline{R}^{\beta}(\emptyset) = \overline{R}^{\beta}(\emptyset) = \emptyset$; $\underline{R}^{\beta}(U) = \overline{R}^{\beta}(U) = U$.
2. $\overline{R}^{\beta}(X) = U - \underline{R}^{\beta}(U - X),\ \forall X \subseteq U$.
3. If $X \subseteq Y \subseteq U$, then

$$\underline{R}^{\beta}(X) \subseteq \underline{R}^{\beta}(Y) \text{ and } \overline{R}^{\beta}(X) \subseteq \overline{R}^{\beta}(Y).$$

4. $\underline{R}^{\beta}(X) \subseteq \overline{R}^{\beta}(X),\ \forall X \subseteq U$.

B. *Belief Functions*

We first recall from [2],

Definition 1. A real-valued function

$$m : 2^{U} \to [0, 1]$$

is called a basic probability assignment, if it satisfies

1) $m(\emptyset) = 0$,

2) $\sum_{E \subseteq U} m(E) = 1$.

A set $E \subseteq U$ with $m(E) > 0$ is referred to as a focal element of $m : 2^{U} \to [0,1]$.

Given a basic probability assignment $m : 2^{U} \to [0, 1]$, according to Shafer [2], the belief and plausibility functions over U, *Bel*: $2^{U} \to [0,1]$ and *Pl*: $2^{U} \to [0,1]$, respectively, are defined as follows: for any $X \subseteq U$,

$$Bel(X) = \sum_{E \subseteq X} m(E), \quad (2.7)$$

$$Pl(X) = \sum_{E \cap X \neq \phi} m(E) \cdot \quad (2.8)$$

The belief and plausibility functions over U are mutually dual in the sense that

$$Pl(X) = 1 - Bel(U - X),\ \forall X \subseteq U. \quad (2.9)$$

We show (2.9) as follows. From Definition 1, (2.7) and (2.8), we obtain

$$\begin{aligned} Pl(X) &= \sum_{E \cap X \neq \varphi} m(E) \\ &= 1 - \sum_{E \subseteq (U - X)} m(E) \\ &= 1 - Bel(U - X). \end{aligned}$$

C. *Incomplete Information Tables*

Definition 2. An information table is a 4-tuple (U, A, V, f), where U is a nonempty finite universe, A is a nonempty finite set of attributes, V is the union of attribute domains, and $f : U \times A \to V$ is an information function defined for every $x \in U$ and $a \in A$, such that $f(x, a) \in V_a$, where V_a is a domain of the attribute a [15].

If V_a contains null value for at least one $a \in A$, the 4-tuple (U, A, V, f), is called an incomplete information table. In what follows, we will denote null value by "*".

III. Main Results

Let $R \subseteq U \times U$ be reflexive, and let

$$Q = \{(x, y) \in U \times U \mid R(x) = R(y)\}.$$

be the so-called derived equivalence relation of R. For $\beta \in [0,0.5)$, using the derived equivalence relation, (2.4) and (2.5) can be rewritten as

$$\begin{aligned} \underline{R}^{\beta}(X) &= \cup \{[x]_Q \mid R(x) \subseteq^{\beta} X\} \\ &= \cup \{[x]_Q \mid 1 - \frac{|R(x) \cap X|}{|R(x)|} \leq \beta\}, \\ &= \cup \{[x]_Q \mid \frac{|R(x) \cap X|}{|R(x)|} \geq 1 - \beta\}, \end{aligned} \quad (3.10)$$

and

$$\begin{aligned} \overline{R}^{\beta}(X) &= \cup \{[x]_Q \mid R(x) \cap^{\beta} X \neq \emptyset\} \\ &= \cup \{[x]_Q \mid 1 - \frac{|R(x) \cap (U - X)|}{|R(x)|} > \beta\} \\ &= \cup \{[x]_Q \mid \frac{|R(x) \cap X|}{|R(x)|} > \beta\}. \end{aligned} \quad (3.11)$$

A. *Incomplete Information Tables and Rough Set Approximations*

Let (U, A, V, f) be an incomplete information table. For each nonempty $B \subseteq A$, define

$$R_B = \{(x, y) \in U \times U \mid \forall a \in B,\ f(x, a) = f(y, a), \text{ or } f(x, a) = *, \text{ or } f(y, a) = *\}. \quad (3.12)$$

As it was shown in Kryszkiewicz [15] that R_B is a reflexive and symmetric relation on the set U. Let

$$Q_B = \{(x, y) \in U \times U \mid R_B(x) = R_B(y)\} \quad (3.13)$$

be the derived equivalence relation of R_B .

In what follows, we shall assume that (U, A, V, f) is an incomplete information table. Let $\beta \in [0,0.5)$. For anynonempty $B \subseteq A$, according to (3.10) and (3.11), we propose a VPGRS model based on the reflexive and symmetric relation R_B on U determined by B as defined in (3.12) as follows:

For $X \subseteq U$,

$$\underline{R_B}^\beta(X) = \cup \{[x]_{Q_B} | R_B(x) \subseteq^\beta X\}$$
$$= \cup \{[x]_{Q_B} | 1 - \frac{|R_B(x) \cap X|}{|R_{B(x)}|} \le \beta\},$$
$$= \cup \{[x]_{Q_B} | \frac{|R_B(x) \cap X|}{|R_{B(x)}|} \ge 1 - \beta\}, \quad (3.14)$$

and

$$\overline{R_B}^\beta(X) = \cup \{[x]_{Q_B} | R_B(x) \cap^\beta X \ne \emptyset\}$$
$$= \cup \{[x]_{Q_B} | 1 - \frac{|R_B(x) \cap (U - X)|}{|R_B(x)|} > \beta\}$$
$$= \cup \{[x]_{Q_B} | \frac{|R_B(x) \cap X|}{|R_{B(x)}|} > \beta\}. \quad (3.15)$$

The above discussion, combined with Lemma 1, leads to the following theorem.

Theorem 1. Let(U, A, V, f) be a given incomplete information table. For each nonempty$B \subseteq A$, define

$$R_B = \{(x, y) \in U \times U \mid \forall a \in B,\ f(x, a) = f(y, a), \text{ or } f(x, a) = *, \text{or } f(y, a) = *\},$$

and let $Q_B = \{(x, y) \in U \times U \mid R_B(x) = R_B(y)\}$ be the derived equivalence relation of R_B. Then R_B is reflexive and symmetric on U. In addition, for any $\beta \in [0,0.5)$, we have the following relations.

1. $\underline{R_B}^\beta(\emptyset) = \overline{R_B}^\beta(\emptyset) = \emptyset$; $\underline{R_B}^\beta(U) = \overline{R_B}^\beta(U) = U$.
2. $\overline{R_B}^\beta(X) = U - \underline{R_B}^\beta(U - X),\ \forall\ X \subseteq U$.
3. If $X \subseteq Y \subseteq U$, then $\underline{R_B}^\beta(X) \subseteq \underline{R_B}^\beta(Y)$ and $\overline{R_B}^\beta(X) \subseteq \overline{R_B}^\beta(Y)$.
4. $\underline{R_B}^\beta(X) \subseteq \overline{R_B}^\beta(X),\ \forall X \subseteq U$.

B. Evidence Theory and VPGRS Model

Let (U, A, V, f) be an incomplete information table. Let

$$R_A = \{(x, y) \in U \times U \mid \forall a \in B,\ f(x, a) = f(y, a), \text{ or } f(x, a) = *, \text{or } f(y, a) = *\}$$

be the reflexive and symmetric relation on U determined by A, and let Q_A be the derived equivalence relation of R_A.

Let

$$F = \{R_A(x) | x \in U\} = \{F_1, F_2, \cdots, F_k\}$$

be the collection of all distinct images of members of U under R_A. For j = 1, 2, …, k, let

$$E_j = \{x \in U \mid R_A(x) = F_j\}.$$

Then $\{E_1, E_2, \cdots, E_k\}$ is the collection of all distinct Q_A-equivalence classes. That is,

$$U/Q_A = \{E_1, E_2, \cdots, E_k\}.$$

Define

$$m_A: 2^U \to [0,1]$$

by assigning

$$m_A(F_j) = \frac{|E_j|}{\sum_{j=1,2,\ldots,k}|E_j|}. \quad (3.17)$$

to each F_j, and zero to all other subsets of U. Then, according to Definition 1, $m_A: 2^U \to [0,1]$ is a basic probability assignment [16].

For any parameter $\beta \in [0,0.5)$,according to (3.14) and (3.15), the β-lower and β-upper approximationsof a set X, $\underline{R_A}^\beta(X)$and $\overline{R_A}^\beta(X)$, respectively, can be rewritten as follows.

$$\underline{R_A}^\beta(X) = \{x \in U \mid R_A(x) \subseteq^\beta X\}$$
$$= \cup \{E_j | F_j \subseteq^\beta X\},$$
$$= \cup \{E_j | 1 - \frac{|F_j \cap X|}{|F_j|} \le \beta\}$$
$$= \cup \{E_j | \frac{|F_j \cap X|}{|F_j|} \ge 1 - \beta\},$$

and

$$\overline{R_A}^\beta(X) = \{x \in U \mid R_A(x) \cap^\beta X \ne \emptyset\}$$
$$= \cup \{E_j | F_j \cap^\beta X \ne \emptyset\}$$
$$= \cup \{E_j | 1 - \frac{|F_j \cap (U - X)|}{|F_j|} > \beta\}$$
$$= \cup \{E_j | \frac{|F_j \cap X|}{|F_j|} > \beta\}. \quad (3.18)$$

According to (3.17) and (3.18), we obtain

$$|\underline{R_A}^\beta(X)| = \sum_{F_j \subseteq^\beta X} |E_j| \quad (3.19)$$

$$\left|\overline{R_A}^\beta(X)\right| = \sum_{F_j \cap^\beta X \ne \emptyset} |E_j|. \quad (3.20)$$

Let us define $Pr: 2^U \to [0,1]$ as follows.

$$Pr(X) = \frac{|X|}{|U|}, \quad \forall X \subseteq U. \quad (3.21)$$

We next define the belief and plausibility functions over U, Bel:$2^U \to [0,1]$ and $Pl: 2^U \to [0,1]$, respectively, as follows. For any $X \subseteq U$,

$$Bel(X) = \sum_{F_j \subseteq^\beta X} m_A(F_j) \quad (3.22)$$

$$Pl(X) = \sum_{F_j \cap^\beta X \ne \emptyset} m_A(F_j) \quad (3.23)$$

Then, according to (3.16)-(3.21), we have

$$Bel(X) = \sum_{F_j \subseteq^{\beta} X} m_A(F_j)$$
$$= \sum_{F_j \subseteq^{\beta} X} \frac{|E_j|}{\sum_{j=1,2,\ldots,k} |E_j|}$$
$$= \frac{1}{\sum_{j=1,2,\ldots,k} |E_j|} \sum_{F_j \subseteq^{\beta} X} |E_j|$$
$$= \frac{1}{|U|} |\underline{R_A}^{\beta}(X)|$$
$$= Pr\left(\underline{R_A}^{\beta}(X)\right),$$

and

$$Pl(X) = \sum_{F_j \cap^{\beta} X \neq \emptyset} m_A(F_j)$$
$$= \sum_{F_j \cap^{\beta} X \neq \emptyset} \frac{|E_j|}{\sum_{j=1,2,\ldots,k} |E_j|}$$
$$= \frac{1}{\sum_{j=1,2,\ldots,k} |E_j|} \sum_{F_j \cap^{\beta} X \neq \emptyset} |E_j|$$
$$= \frac{1}{|U|} \left|\overline{R_A}^{\beta}(X)\right|$$
$$= Pr\left(\overline{R_A}^{\beta}(X)\right).$$

We summarize the results of this discussion in the following theorem.

Theorem 2. Considering an incomplete information table (U, A, V, f), let

$$R_A = \{(x, y) \in U \times U \mid \forall\, a \in A,\ f(x, a) = f(y, a), \text{ or } f(x, a) = *, \text{ or } f(y, a) = *\}$$

be the reflexive and symmetric relation on U determined by A, and let Q_A be the derived equivalence relation of R_A. Let

$$F = \{R_A(x) \mid x \in U\} = \{F_1, F_2, \cdots, F_k\}$$

be the collection of all distinct images of members of U under R_A, and let

$$E_j = \{x \in U \mid R_A(x) = F_j\}.$$

For j=1, 2, …, k, let

$$m_A(F_j) = \frac{|E_j|}{\sum_{j=1,2,\ldots,k} |E_j|}$$

and let $m_A(S) = 0$ for all other subsets $S \subseteq U$. Then

$$m_A: 2^U \to [0,1]$$

is a basic probability assignment, and the belief and plausibility functions over U,

$$Bel: 2^U \to [0,1] \quad \text{and} \quad Pl: 2^U \to [0,1],$$

can be defined as follows.

$$Bel(X) = \sum_{F_j \subseteq^{\beta} X} m_A(F_j)$$
$$= Pr\left(\underline{R_A}^{\beta}(X)\right),$$

and

$$Bel(X) = \sum_{F_j \cap^{\beta} X \neq \emptyset} m_A(F_j)$$
$$= Pr\left(\overline{R_A}^{\beta}(X)\right)$$

for any $X \subseteq U$.

IV. An Illustrative Example

For the sake of illustration, we present in this section an example of an incomplete information table (U, A, V, f), which is shown in Table I. From this table, we have

$$U = \{1, 2, 3, 4, 5, 6\},\ A = \{p, q\}.$$

Let R_A be the reflexive and symmetric relation on U determined by A as defined in (3.12). According to Table I, the images are:

$$R_A(1) = R_A(2) = \{1, 2, 6\},\ R_A(3) = \{3\},$$
$$R_A(4) = R_A(5) = \{4, 5, 6\},\ R_A(6) = \{1, 2, 4, 5, 6\}.$$

From the images of R_A, we assume that

$$F_1 = \{1, 2, 6\},\ F_2 = \{3\},$$
$$F_3 = \{4, 5, 6\},\ F_4 = \{1, 2, 4, 5, 6\}.$$

It follows that

$$E_1 = \{1, 2\},\ E_2 = \{3\}, E_3 = \{4, 5\},\ E_4(6) = \{6\}.$$

TABLE I: Example of an Incomplete Information Table

U	p	q
1	1	1
2	1	1
3	2	2
4	1	2
5	1	2
6	1	*

Let Q_A be the derived equivalence relation of R_A, then we have

$$U/Q_A = \{\{1,2\}, \{3\}, \{4,5\}, \{6\}\}.$$

Let us approximate the sets $X = \{1, 2, 3\}$, and $Y = \{1, 2, 3, 4, 6\}$ for the threshold $\beta = 0.3$.

According to (2.3), we have

$$e(F_1, X) = 1 - \frac{2}{3} = \frac{1}{3}, \qquad e(F_2, X) = 1 - \frac{1}{1} = 0,$$
$$e(F_3, X) = 1 - \frac{0}{3} = 1, \qquad e(F_4, X) = 1 - \frac{2}{5} = \frac{3}{5},$$
$$e(F_1, U - X) = \frac{2}{3}, \qquad e(F_2, U - X) = 1,$$
$$e(F_3, U - X) = 0, \qquad e(F_4, U - X) = \frac{2}{5},$$
$$e(F_1, Y) = 1 - \frac{3}{3} = 0, \qquad e(F_2, Y) = 1 - \frac{1}{1} = 0,$$
$$e(F_3, Y) = 1 - \frac{2}{3} = \frac{1}{3}, \qquad e(F_4, Y) = 1 - \frac{4}{5} = \frac{1}{5},$$

According to (3.16), we have

$$m_A(F_1) = \frac{1}{3},\ m_A(F_2) = \frac{1}{6},$$

$$m_A(F_3) = \frac{1}{3},\ m_A(F_4) = \frac{1}{6}$$

Therefore, we have

$$\underline{R_A}^{0.3}(X) = E_2 = \{3\}$$

$$\overline{R_A}^{0.3}(X) = E_1 \cup E_2 \cup E_4 = \{1, 2, 3, 6\}$$

$$\underline{R_A}^{0.3}(U - X) = E_3 = \{4, 5\}$$

$$\overline{R_A}^{0.3}(U - X) = E_1 \cup E_3 \cup E_4 = \{1, 2, 4, 5, 6\}$$

$$\underline{R_A}^{0.3}(Y) = E_1 \cup E_2 \cup E_4 = \{1, 2, 3, 6\}$$

$$\overline{R_A}^{0.3}(Y) = E_1 \cup E_2 \cup E_3 \cup E_4 = \{1, 2, 3, 4, 5, 6\}$$

According to (3.22) and (3.23), we obtain

$$\begin{aligned}
Bel(X) &= \textstyle\sum_{F_j \subseteq^{\beta} X} m_A(F_j) \\
&= m_A(F_2) \\
&= \tfrac{1}{6} = Pr\left(\underline{R_A}^{0.3}(X)\right) \\
&= 1 - Pr\left(\overline{R_A}^{0.3}(U - X)\right) \\
&= 1 - Pl(U - X)
\end{aligned}$$

$$\begin{aligned}
Pl(X) &= \textstyle\sum_{F_j \cap^{\beta} X \neq \emptyset} m_A(F_j) \\
&= m_A(F_1) + m_A(F_2) + m_A(F_4) \\
&= \tfrac{1}{3} + \tfrac{1}{6} + \tfrac{1}{6} \\
&= Pr\left(\overline{R_A}^{0.3}(X)\right) \\
&= 1 - Pr\left(\underline{R_A}^{0.3}(U - X)\right) \\
&= 1 - Bel(U - X)
\end{aligned}$$

$$\begin{aligned}
Bel(Y) &= m_A(F_1) + m_A(F_2) + m_A(F_4) \\
&= \tfrac{1}{3} + \tfrac{1}{6} + \tfrac{1}{6} = Pr\left(\underline{R_A}^{0.3}(Y)\right)
\end{aligned}$$

$$\begin{aligned}
Pl(Y) &= m_A(F_1) + m_A(F_2) + m_A(F_3) + m_A(F_4) \\
&= \tfrac{1}{3} + \tfrac{1}{6} + \tfrac{1}{3} + \tfrac{1}{6} = Pr\left(\overline{R_A}^{0.3}(Y)\right)
\end{aligned}$$

This example validates the results in Section III.

V. Conclusion

We connect evidence theory, rough set theory and the variable precision concept to present applications in incomplete information tables. More precisely, for a given parameter, we extend Pawlak's lower and upper approximations to set approximations in the VPGRS models. We also extend the relationship between lower and upper approximations from VPGRS models to incomplete information tables. We further show the duality between the belief and plausibility functions in evidence theory. It shows potentials for more applications. The implications of this paper are to release the limitation of rough set theory. So, one can use more tools to deal with problems with incomplete information tables. We will work out more examples and case studies in a future project.

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