

# Effect of Drop and Rebuilt Operator for Solving the Biobjective Obnoxious $p$ -Median Problem

Méziiane Aïder, Aida-Ilham Azzi, and Mhand Hifi\*

**Abstract**—In this paper, we solve the bi-objective obnoxious with a population-based method. The designed algorithm first determines a starting archive set by applying an iterative search on the equivalent problem, where an aggregate function is considered. Second, an adaptation of the dominating local search, combined with exchange operators, is considered for generating a series of new non-dominated solutions that enrich the reference archive set. Third, a drop and rebuild strategy is incorporated to the algorithm for iteratively highlighting the final Pareto front. An experimental part is given, where the performance of the method is evaluated on a set of benchmark instances of the literature. Its provided results are compared to those achieved by the more recent methods available in the literature. Encouraging results are reached.

**Index Terms**—Bi-objective, heuristics, obnoxious, optimization.

## I. INTRODUCTION

In this paper, we approximately solve the Bi-Objective Obnoxious  $p$ -Median Problem (Bi-OpM). Such a problem is NP-hard and it has been first studied in Church and Garfinkel [1], and Erkut and Neuman [2]. The problem deals with situations with facilities presenting obnoxious or semi-obnoxious features. Often an obnoxious induces a negative (dangerous) influence regarding the surrounding area, and such a type of facility may be involved when dealing with several manipulations related to hazardous materials, waste disposal, water treatment and others.

Herein, we focus on optimizing both the sum of the minimum distance between each customer and its nearest open facility, and the maximum dispersion between facilities. This problem has a direct application in locations of hazardous facilities, like nuclear or chemical power plant waste storage facilities and noisy or polluting services Colmenar *et al.* [3]. An instance of Bi-OpM may be defined as follows: let  $I$  be a set of clients,  $J$  a set of facilities such that a subset of  $p$  locations should be determined; that is a subset optimizing two objective functions: (i) maximizing the sum of the minimum distance between each customer and its nearest open facility  $dc_{ij}$  and (ii) maximizing the dispersion between facilities  $df_{j_1j_2}$ , where  $j_1, j_2 \in S \times S \subseteq J$  and  $j_1 < j_2$ . Facilities belonging to  $S$  denote the open facilities while those contained in  $J \setminus S$  represent the closed or non-open facilities.

The paper is organized as follows. Section II reviews

the relevant literature on some Obnoxious problems. Section III-A provides a formal description of the problem studied. Section III-B discusses the dominated local search used to approximately solve Bi-OpM. A starting archive of feasible solutions is described in Section III-C. Section III-F discusses local operators that are based upon drop and rebuild operator, where it is applied for highlighting the density of the archive set. The performance of the proposed method is exposed in Section IV. Finally, Section V concludes the paper.

## II. BACKGROUND

Early work on Obnoxious  $p$ -Median (OpM) problem was first discussed in Belotti *et al.* [4], where a branch-and-cut algorithm has been designed. This method is mainly based upon three families of valid inequalities for which the authors demonstrated their performances when considered some particular instances. A more detailed description of OpM problem was discussed in Colmenar *et al.* [5], where the authors underlined its goal for maximizing the sum of minimum distances between each open facility and its nearest client. Their method is based on the adaptation of GRASP. Lin and Guan [6] proposed a hybrid binary particle swarm optimization for tackling OpM problem. Their method is based on the position updating rule combined with a tabu-based mutation operator; that is a strategy used for generating good diversified solutions. The algorithm was augmented with a greedy procedure for repairing infeasible solutions.

Gokalp [7] developed an iterated greedy metaheuristic for approximately solving OpM problem. The method combines two fast and efficient local search procedures. Herran and Colmenar *et al.* [8] tried to adapt the variable neighborhood search for tackling the same problem. The proposed algorithm integrates two local search procedures that are based upon the reduced neighborhood exploration such that a balance between solution quality and runtime was favored. Chang and Wanga *et al.* [9] solved OpM problem with a parallel iterative solution-based tabu search, where the tabu search was used to prevent cycling and stagnation of solutions.

Regarding the multi-objective version of OpM problem, Ardjmand and Young *et al.* [10] proposed a genetic algorithm for dealing with a special case related to hazardous materials. In their work, transportation costs are stochastic such that the first objective function minimizes the total cost and risk of locating facilities, and the second one optimizes the transportation of hazardous materials. In the experimental part, the behavior of their proposed method was analyzed and its provided results were

Manuscript received June 30, 2021; revised July 22, 2021; accepted August 12, 2022.

M. Aïder and A-I. Azzi are with LaROMaD, Fac. Maths, USTHB, BP 32, Bab Ezzouar, 16111, Algeria.

Mhand Hifi is with EPROAD UR 4669, UPJV, Amiens, France.

\*Correspondence: hifi@u-picardie.fr

compared to those achieved by an exact method. Encouraging results were obtained.

Coutinho-Rodrigues and Tralhao [11] studied another special case related to the urban planning problem such that objective functions consist of minimizing the total investment cost, and minimizing the weighted average customer dissatisfaction. A first mixed-integer bi-objective programming approach, to identify the locations and capacities of semi-desirable (or semi-obnoxious) facilities, was designed.

In this work, we propose a first alternative hybrid population-based method for efficiently solving the bi-objective Obnoxious  $p$ -median problem, where both (i) the sum of the minimum distance between each customer and its nearest open facility, and (ii) the maximum dispersion between facilities, are optimized.

### III. THE BI-OBJECTIVE OBNOXIOUS $P$ -MEDIAN

In this work, the Bi-Objective Obnoxious  $p$ -Median Problem (Bi-OpM), a NP-hard combinatorial optimization problem is studied. We mainly focus on optimizing two objectives.

#### A. The Model

A formal description of Bi-OpM can be stated as follows (cf., Colmenar and Martí *et al.* [3]):

$$\begin{aligned} \max z_1 &= \sum_{i \in I} \min_{j \in J} \{dc_{ij}\} \\ \max z_2 &= \sum_{j_1 \in J} \min_{j_2 \in J} \{df_{j_1 j_2}, j_1 < j_2\} \\ \text{Subject to} \quad & S \subseteq J \text{ and } |S| = p, \end{aligned}$$

where  $S, S \subseteq J$  (of cardinality  $p$ ) should be determined such that  $z_1$  refers to the first objective function which maximize the sum of the minimum distance between each customer and its nearest open facility  $dc_{ij}$ ,  $i \in I$  and  $j \in J$  and  $z_2$  expresses the second objective function such that for each pair of facilities  $(j_1, j_2) \in S \times S$  with  $S \subseteq J$ , their related dispersion  $df_{j_1 j_2}$  should be maximized.

#### B. Dominance Local Search

Local search-based methods have been already designed for single objective combinatorial optimization problems. By introducing the dominance criteria related to the problems with multiple objective functions, these methods can be tailored for tackling several multi-objective combinatorial optimization problems. A classical local search is often based on an iterative search, where enhancing the quality of solutions is realized throughout an optimization process. In this case, the search procedure iteratively explores one or several neighborhoods related to a current solution hopping to converge toward local optimum. Thus, each neighborhood structure should be defined for better exploring the whole search space of feasible solutions. In order to adapt such a search process to a problem with many objective functions, the following Pareto dominance rule is considered.

**Dominance rule:** For a given maximization problem

with objective functions  $z_1, z_2, \dots, z_m$ , a solution  $x^{(1)}$  dominates a solution  $x^{(2)}$  (noted  $x^{(1)} \succ x^{(2)}$ ) if and only if inequalities hold:

1.  $\forall i \in \{1, \dots, k\}; z_i(x^{(1)}) \geq z_i(x^{(2)})$ ,
2.  $\exists i \in \{1, \dots, k\}; z_i(x^{(1)}) > z_i(x^{(2)})$ .

A feasible solution  $x^*$  is a Pareto optimal solution if  $\nexists y \in \Omega$  such that  $z(y) \succ z(x^*)$ . The set of all Pareto optimal solutions is called the Pareto-optimal set  $\mathcal{P}$ , where  $\mathcal{P} = \{x^{(1)} \in \Omega \mid \nexists y \in \Omega \text{ such that } z(y) \succ z(x^*)\}$ . Hence, by applying the dominance rule, we then establish the set of Pareto optimal solutions  $\mathcal{P}$  of non-dominated solutions.

#### C. A Starting Archive Set

First, the archive set, representing an approximate Pareto front, is built by applying a deterministic greedy adaptive search procedure. We do it by a linear scalarizing of the bi-objective optimization problem, where any (optimal) solution related to the single-objective optimization problem is a Pareto solution to the bi-objective optimization problem. The single function using a linear scalarization of Bi-OpM may be established as follows:

$$z_{ij} = \omega \times z_1 + (1 - \omega) \times z_2 \quad (1)$$

where,  $\omega$  represents a weight such that  $\omega \in [0, 1]$ , and  $z_1 = dc_{ij}$  and  $z_2 = \min_{j \in J} \{df_{j_1 j_2}\}$ . One can observe that by varying the weight  $\omega$  in Eq. (1) and by solving each of these induced optimization problems it may induce an exploration of several complementary search sub-spaces. From these resolutions, a starting archive set, namely  $\mathcal{A}$  of diversified solutions may also be reached.

Second, at each iteration of the generation process,  $F = (F_1, \dots, F_{|J|})$  is sorted in decreasing order of its values  $F_j$ ,  $j \in J$ , and  $p$  facilities are picked according to the first  $p$  largest values. One can observe that the aforementioned selected facilities implies a feasible solution for Bi-OpM at the current iteration such that

$$F_j = \sum_i z_{ij}, i = 1, \dots, |I|. \quad (2)$$

The provided solution is then added to the archive set  $\mathcal{A}$  (initially, this set  $\mathcal{A}$  was setting equal to an empty set). Of course, one can observe that by updating the weight  $w$  with a value  $\alpha$  such that  $0 \leq \alpha \leq 0.1$ , a series of feasible solutions can be built and so, the archive set can be updated with these solutions. We note that whenever the archive set growth, the non-dominated solutions are favoured.

Algorithm 1 describes the main steps of the generation procedure, where a starting archive set  $\mathcal{A}$  is built. The algorithm initializes (line 1) the archive set  $\mathcal{A}$  to an empty set and  $\omega$  to zero (the parameter Iter is used for controlling the stopping condition). The algorithm is composed of a global loop (lines from 2 to 9) that is used for building a series of solutions according to the scalarization function with its current weight  $\omega$ . First, the single objective function is modified at line 3 with the current weight  $\omega$ , and the vector  $F$  related to all facilities is computed at line 4. Second, a feasible solution  $P$  is provided by combining lines 5 and 6, where  $F$  is sorted in decreasing order of its values and  $P$  is that achieving the best  $p$  greatest values of  $F$ . Third, the

archive set  $\mathcal{A}$  is updated with  $P$  (line 7), a new weight for the next single objective problem is generated (line 8) and the parameter  $\text{Iter}$  is updated. These steps are iterated till matching the stopping condition; that is limited to  $\frac{1}{\alpha}$ , where  $\alpha$  is a given parameter belonging to the interval  $[0, 0.1]$ .

---

**Algorithm 1** A starting archive set
 

---

**Input:** An instance of Bi-OpM.  
**Output:** An archive set  $\mathcal{A}$ .  
 1: Set  $\text{Iter} = 1$ ,  $\omega = 0$  and  $\mathcal{A} = \emptyset$ .  
 2: **repeat**  
 3:   Set  $z_{ij} = \omega \times z_1 + (1 - \omega) \times z_2, \forall i \in I$  and  $\forall j \in J$ .  
 4:   Let  $F = (F_1, \dots, F_j, \dots, F_{|J|})$  such that  
        $F_j = \sum_j f_{ij}, \forall j \in J$ .  
 5:   Sort  $F$  in decreasing order of its values such that  $F_1 \geq F_2 \geq \dots \geq F_{|J|}$ .  
 6:   Let  $P$  be a feasible solution represented by the indices of the  $p$  best values of  $F$ .  
 7:   Update  $\mathcal{A}$  with  $P$ .  
 8:   Set  $\omega = \omega + \alpha$  and  $k = k + 1$ .  
 9: **until**  $(k > \frac{1}{\alpha}) // 0 < \alpha \leq 0.1$   
 10: **return**  $\mathcal{A}$ .

---

#### D. Improving Operators

Often the operators used by classical local searches are mainly based on k-opt procedures, where both 2-opt and 3-opt are the most popular procedures. Herein, we adapt both operators to improve the set of the archive set  $\mathcal{A}$  even if it is based upon simple moves. In this case we apply the exchange strategy.

- 1) *1-exchange operator*: Let  $\hat{P}$  be a feasible solution and  $S$  denote the  $p$  facilities forming  $\hat{P}$  and  $\bar{S}_p = J \setminus S_p$  be the set of unselected facilities. The 1-exchange operator iteratively makes swaps as long as the dominance criterion is satisfied. Herein, two fixed chosen positions  $i$  and  $j$ , such that  $i \in S_p$  and  $j \in \bar{S}_p$ , are swapped. Next, a new assignment is reached that represent a feasible solution for Bi-OpM. We note that iterating such a process, by using two loops, induces a neighborhood around the solution  $\hat{P}$ .
- 2) *Avoid cycling*: The exchange operator builds a series of solutions that are iteratively reached throughout searching on a number of neighborhoods. In order to avoid cycling, the so called tabu list is added to store some exchanges instead of storing all visited solutions. Indeed, the list saves temporary some inverse -exchanges for avoiding stagnation of solutions. A tabu list is then added such that for each explored solution, the size of the tabu list was fixed to  $\min\{p, n - p\}$  such that a FIFO strategy is used.
- 3) *2-exchange operator*: E2: For a given solution  $\hat{P}$ , instead of using a couple of positions belonging to different sets  $S_p$  and  $\bar{S}_p$ , we extend the exchanges to four positions which provide a 2-exchange operator (E2). Indeed, for different positions  $i, j, k$  and  $l$ , the first exchange between  $i \in S_p$  and  $j \in \bar{S}_p$  generates a new solution, and the second exchange between  $k$  and  $l$  induces a new solution. Of course, by using such a process, we also avoid cycling by using the same tabu list, as described in Section III-D2.

#### E. Drop and Rebuild Operator

For a given solution  $\hat{P}$ , instead of using a couple of positions (1-exchange) belonging to different sets  $S_p$  and  $\bar{S}_p$ , one can observe that making a diversification on  $\hat{P}$  can be

provided by driving the search process throughout unvisited search subspaces. We do it by extending 1-exchange to  $\beta$ -exchanges such that  $\beta\%$  of positions belonging to  $S_p$  is dropped and the  $\beta\%$  of the removed position are replaced with those of  $\bar{S}_p$ .

---

**Algorithm 2** Drop and rebuild operator
 

---

**Input:** An instance of Bi-OpM with a current solution  $\hat{P}$ .  
**Output:** A non-dominated set  $\mathcal{B}$  of solutions.  
 1: Set  $P = \hat{P}$  and  $\mathcal{B} = \emptyset$ .  
 2: **repeat**  
 3:   Randomly drop  $\beta\%$  of facilities from  $S_p$ .  
 4:   By using a tabu list, rebuild a complete solution  $P^*$  by adding facilities belonging to  $\bar{S}_p$  according to the decreasing order of  $F(P) = (F(P)_1, \dots, F(P)_{n-p})$ .  
 5:   Call 1-exchange for enhancing  $P^*$  according to  $z_1$  and  $z_2$ .  
 6:   Let  $P^*$  the best resulting solution when compared to  $P$ .  
 7:   Update  $\mathcal{B}$  with  $P^*$  and set  $P = P^*$  (whenever  $P^* \notin \mathcal{B}$ ).  
 8: **until** (a predefined condition is performed)  
 9: Purge  $\mathcal{B}$ .  
 10: **return**  $\mathcal{B}$ .

---

Of course, by using such a process, we also avoid cycling by using the same tabu list, as described in Section III-D2. In order to mimic DR operator, the following steps are given:

Let  $\hat{P}$  be the solution at hand.

Randomly drop  $\beta\%, \beta \in [0, 100]$ , of facilities from  $S_p$  such that  $\beta \leq \frac{\min\{p, n-p\}}{100}$ .

Let  $\hat{P}$  be the new solution built by randomly adding  $\beta\%$  of facilities belonging to  $\bar{S}_p$ . In this case, a tabu list is used for avoiding cycling, especially when making exchanges between a series of couple of items (facilities).

One can observe that a random choice of facilities belonging to both sets may provide an aggressive diversification for the current solution  $\hat{P}$ , which in some cases the quality of the solution may be degraded according to both objective function of Bi-OpM. Hence, in order to enhance the quality of the solution reached, a modified 1-exchange operator is proposed, in which the greedy procedure using the best values of  $F$  is also considered.

Algorithm 2 summarizes the main steps of the drop and rebuild operator used for enhancing the solution  $\hat{P}$  at hand.

#### F. An Overview of the Population-Based Method with Drop and Rebuilt Operator for Bi-OpM

Algorithm 3 describes the main steps of the population-based method with Drop and Rebuilt Operator (denoted PBM with D&R), where this operator is used as a learning strategy. The input of PBM with D&R is an instance of Bi-OpM and its output is an archive set of (approximate Pareto) optimal solutions  $\mathcal{A}$ . It starts with a starting archive set (line 10) built by calling Algorithm 1.

The method contains three loops: a global loop and two internal loops. The global loop “repeat” from line 3 to line 22 explores a subset (or all) solutions of the starting archive set  $\mathcal{A}$  (built with Algorithm 3: line 1). The first internal loop “repeat” from line 7 to line 13 searches for a better solution around the current one, where the non-dominated solutions are preferred. In this case, a random 1-exchange operator is applied for providing a series of intensified solutions. Next, the DR operator is called whenever the solution stagnates (or a number of local iterations is performed). Thus, the second local loop (line 5 to line 21) is restarted whenever a

new solution (improving either  $z_1$  or  $z_2$ ) is reached; the algorithm selects a new solution from  $\mathcal{A}$  and so, till matching final conditions otherwise. Finally, the algorithm exits with an approximate Pareto set  $\mathcal{A}$ .

**Algorithm 3** - A Population-Based Method with Drop and Rebuilt Operator (PBM with D&R)

---

Input. A instance of Bi-OpM.  
Output. A final archive set  $\mathcal{A}$ .

---

```

1: Call Algorithm 1 for generating the starting archive set  $\mathcal{A}$ .
2: Set stop=false
3: while (either a subset of solutions of  $\mathcal{A}$  are visited or a maximum number of
   iterations is matched) do
4:   Choose  $\hat{P}$  from the archive set  $\mathcal{A}$  and mark it as visited.
5:   repeat
6:     Set  $Iter_{local}=0$ 
7:     repeat
8:       Apply a random 1-exchange operator (with avoiding cycling).
9:       Let  $\hat{P}'$  be the new solution built; Increment  $Iter_{local}$ .
10:      if ( $\hat{P}'$  is a non-dominated solution) then
11:        Update the archive set  $\mathcal{A}$  with  $\hat{P}'$ .
12:      end if
13:    until ( $Iter_{local}$  matches a number of local iterations)
14:    Call Algorithm 2 with the best  $\hat{P}'$  (favoring  $z_1$ ) and let  $\mathcal{B}$  be the resulting
    non-dominated set.
15:    if ( $\exists$  new non-dominated solutions belonging to  $\mathcal{B}$ ) then
16:      Update the archive set  $\mathcal{A}$  with the non-dominated solutions of  $\mathcal{B}$ .
17:      Set  $Iter_{local}=0$ 
18:    else
19:      stop=true
20:    end if
21:  until (stop)
22: end while
23: Purge  $\mathcal{A}$  by keeping the non-dominated solutions.
24: return  $\mathcal{A}$ .
```

---

#### IV. EXPERIMENTAL PART

Herein, a preliminary experimental part is given for evaluating the behavior of the proposed Population-Based Method (PBM with D&R). Its performance is evaluated on a set of benchmark instances extracted from [3], [12], where it's provided results are compared to those archived by the best methods available in the literature.

We note that the proposed method was coded in C++ and performed on a computer with an Intel Pentium Core i5 with 2.80 GHz. In our preliminary tests, we considered 8 large-scale instances over the 72 existing ones, where the number of nodes varies from 400 to 900, the number of clients and facilities ( $|I|$  and  $|J|$ ) vary from 200 to 450, and the number of open facilities related to  $p$  belongs to the discrete interval [25, 225].

TABLE I: EFFECT OF THE PARAMETER  $\alpha$  USED BY ALGORITHM 1

$\alpha$	#Sol	HV	Best $z_1$	Best $z_2$	CPU (s)
$\alpha = 0.1$	9.38	0.866	5480.38	1883.8750	2.33
$\alpha = 0.01$	33.88	0.987	5485.25	1890.1250	18.65
$\alpha = 0.01$	<b>41.13</b>	<b>1.000</b>	<b>5487.13</b>	<b>1890.1250</b>	<b>208.15</b>

##### A. Parameter Settings

Often heuristics may lead results of variable quality, especially when plenty parameters are used for solving complex combinatorial optimization problems. The designed algorithm (PBM with D&R) needs two decision parameters: (i) the number of items to remove related to the DR operator, and (ii) the value related to  $\alpha$  that is used by Algorithm 1 for achieving the starting archive set  $\mathcal{A}$ .

Table I reports the results provided by the PBM with D&R when fixing  $\alpha$  to 0.1, 0.01 and 0.001 (the variation of  $\alpha$  is represented in column 1). Column 2 of the table

displays the average number of non-dominated solution belong to the approximate Pareto set  $\mathcal{A}$ . column 3 tallies the average normalized hypervolume obtained by PBM with D&R when varying  $\alpha$ , column 4 (resp. column 5) reports the average bound related to  $z_1$  (resp.  $z_2$ ), and column 6 displays the average runtime needed for achieving the final archive set.

From Table I, one can observe that all indicators used are favourable for  $\alpha = 0.01$ , except for the average runtime which growth because of the size of the starting archive set  $\mathcal{A}$ .

TABLE II: VARIATION OF THE NORMALIZED HYPERVOLUME VALUE ON SOME INSTANCES OF THE LITERATURE

#Inst	Mo-PVNS	AOLS	DBLS	NSGAII	SPEA2	This work PBM with D&R
pmed17.p25	0.9682	0.9992	<b>1.0000</b>	0.9986	0.9996	0.8538
pmed 20.p50	0.7370	0.7292	0.7276	0.6155	0.7153	<b>0.9982</b>
pmed 22.p62	<b>1.0000</b>	0.9719	0.6936	0.6745	0.9144	0.9971
pmed 28.p75	<b>1.0000</b>	0.9583	0.9502	0.6277	0.8190	0.8820
pmed 33.p87	0.8632	0.7480	0.7433	0.3722	0.5494	<b>0.9996</b>
pmed 36.p100	<b>1.0000</b>	0.8177	0.8246	0.3460	0.5334	0.9630
pmed 38.p112	<b>1.0000</b>	0.8206	0.8204	0.3279	0.5138	0.7896
pmed 40.p225	0.8622	0.6253	0.6367	0.1960	0.2709	<b>0.9855</b>
Average	0.9288	0.8338	0.7995	0.5198	0.6645	<b>0.9336</b>

##### B. Quantitative Study

For multi-objective optimization problems, there are several performance indicators dedicated to analyzing the behavior of a given method. In this part, we focus on the *hypervolume Indicator* that remains one of the principal indicators used in the literature. In this case, the proposed method PBM with D&R is compared to the best methods of the literature (cf., Colmenar and Mart *et al.* [3]): Mo-PVNS, AOLS, DBLS, NSGA-II and SPEA2. Their average normalized hypervolume indicators are reported in Table II: column 1 reports the instance's label, columns from 2 to 6 tally the average normalized hypervolume indicator of Mo-PVNS, AOLS, DBLS, NSGA-II and SPEA2, respectively while column 7 reports PBM's with D&R average normalized hypervolume indicator. The last line of the table summarizes the average global values related to all tested instances.

From Table II, we observe that PBM's with D&R global average value (last line of the table, column 7) is better (0.93360) than the maximum value provided by all other methods (0.92883). Further, PBM with D&R is able to three better normalized hypervolume when compared to more complex methods. We believe that PBM with D&R is good candidate for hybridization with an evolutionary population algorithm. In order to evaluate the behavior of PBM with D&R, we also considered the net front contribution indicator of each method. For two algorithm A and B such that  $A_A$  and  $A_B$  denote their archive sets, the net front contribution  $\overline{NFC}(A_A; A_B)$  is the subset  $A_A$  of belonging to  $A_A \cup A_B$ . Table III reports the results of PBM with D&R and the other five methods. From this table, one can observe that PBM with D&R is able to archive a better net front contribution for all tested instances (varying from 0.2256- instance pmed20.p50, to 0.2886-instance pmed39.p112) with a global average value of 0.2595 (last line of the table, column

7).

Finally, Fig. 1 and Fig. 2 illustrate the density of the Pareto front achieved by the six methods for two instances: inst-pmed20.p50 (Fig. 1) and inst-pmed40.p225 (Fig. 2). One can observe that for both instances, the density of the Pareto front is much better than those provided by the five other methods.

TABLE III: THE NET FRONT CONTRIBUTION INDICATOR: PBM WITH D&R VRSUS THE FIVE OTHER METHODS

#Inst	Mo-PVNS	AOLS	DBLS	NSGAI	SPEA2	PBM with D&R
pmed17.p25	0.1137	0.1031	0.1031	0.0960	0.0829	<b>0.2476</b>
pmed 20.p50	0.1977	0.1020	0.1056	0.0451	0.0930	<b>0.2256</b>
pmed 22.p62	0.2162	0.1060	0.1260	0.0142	0.0434	<b>0.2462</b>
pmed 28.p75	0.2276	0.0860	0.0964	0.0113	0.0287	<b>0.2719</b>
pmed 33.p87	0.2575	0.0777	0.0867	0.0235	0.0271	<b>0.2620</b>
pmed 36.p100	0.2456	0.0667	0.0943	0.0125	0.0142	<b>0.2838</b>
pmed 38.p112	0.2558	0.0728	0.0844	0.0151	0.0062	<b>0.2886</b>
pmed 40.p225	<b>0.3265</b>	0.0914	0.1027	0.0184	0.0198	0.2500
Av	0.2301	0.0882	0.0999	0.0295	0.0394	<b>0.2595</b>

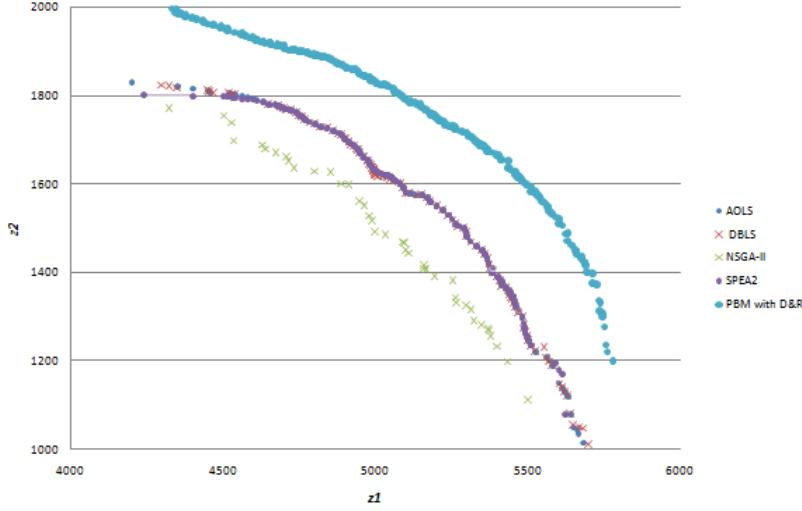


Fig. 1. Illustration of the approximate Pareto front achieved by the five Methodson”inst-pmed20.p50”.

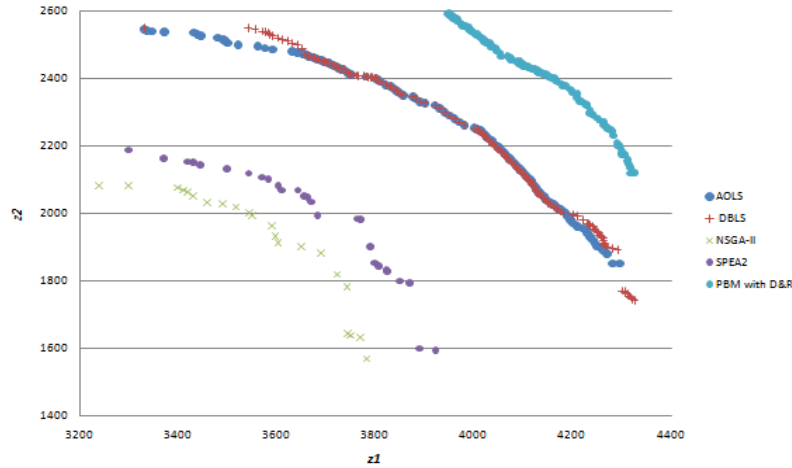


Fig. 2. Illustration of the approximate Pareto front achieved by the five methods on “inst-pmed40.p225”.

## V. CONCLUSION

In this paper, we solved the bi-objective with a population-based method. The method is based upon the so-called non-dominating operator, where both the sum of the minimum distances between each customer and its nearest open facility, and the maximum dispersion between facilities are optimized. First, a starting archive set of solutions was built by tailoring a constructive greedy procedure based upon the scalarization of the bi-objective version of the problem. Second intensification and diversification based on drop and rebuild operator, augmented with a complementary procedure, were introduced for enriching the non-dominated archive set. The preliminary computational part showed that the designed method remains competitive when compared its achieved results to those provided by the best methods available in the literature. For a future work, we are looking for the

hybridization between evolutionary algorithms and drop/rebuild operator.

## CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

## AUTHOR CONTRIBUTIONS

Mhand Hifi proposed the main idea related to this work. He designed the original bi-objective dominated local search combined with the drop and build strategy. He supervised all the work of this paper.

Ilham Azzi (PhD student supervised by Méziane Aïder) coded the method which is based upon the above idea. She tested the final version on a set of benchmark instances.

Mhand Hifi investigated the sensitivity analysis, extended the experimental part with a statistical analysis, and write the first version of the paper.

All authors provided the final version of the paper.



## REFERENCES

- [1] R. Church and R. Garfinkel, "Locating an obnoxious facility on a network. *Transportation Science*, vol. 12, no. 2, pp. 107–118, 1978.
- [2] E. Erkut and S. Neuman, "Analytical models for locating undesirable facilities," *European Journal of Operations Research*, vol. 40, no. 3, pp. 275–291, 1989.
- [3] J. M. Colmenar, R. Martí and A. Duarte, "Multi-objective memetic optimization for the obnoxious p-median problem," *Knowledge-Based Systems*, vol. 144, pp. 88–101, 2018.
- [4] P. Belotti, M. Labbé, F. Maffioli, and M. Ndiaye, "A branch-and-cut method for the obnoxious p-median problem," *A Quarterly Journal of Operations Research*, vol. 5, no. 4, pp. 299–314, 2007.
- [5] J. M. Colmenar, P. Greistorfer, R. Martí and A. Duarte, "Advanced greedy randomized adaptive search procedure for the obnoxious p-median problem," *European Journal of Operations Research*, vol. 252, no. 2, pp. 432–442, 2016.
- [6] G. Lin and J. Guan, "A hybrid binary particle swarm optimization for the obnoxious p-median problem," *Information Sciences*, vol. 425(C), pp. 1–17, 2018.
- [7] O. Gokalp, "An iterated greedy algorithm for the obnoxious p-median problem," *Engineering Applications of Artificial Intelligence*, vol. 92, pp. 103–674, 2020.
- [8] Herran, J. M. Colmenar, R. Martí and A. Duarte, "A parallel variable neighborhood search approach for the obnoxious p-median problem," *International Transactions in Operational Research*, vol. 27, pp. 336–360, 2018.
- [9] J. Chang, L. Wanga, J. K. Hao, and Y. Wang, "Parallel iterative solution based tabu search for the obnoxious p-median problem," *Computers & Operations Research*, vol. 127, 2021.
- [10] E. Ardjmand, W. A. Young, G. R. Weckman, O. S. Bajgiran, B. Aminipour, and N. Park, "Applying genetic algorithm to a new bi-objective stochastic model for transportation, location, and allocation of hazardous materials," *Expert Systems with Applications*, vol. 58, pp. 51–49, 2016.
- [11] J. Coutinho-Rodrigues, L. Tralhão, and L. Alçada-Almeida, "A biobjective modeling approach applied to an urban semi-desirable facility location problem," *European Journal of Operations Research*, vol. 223, no. 1, pp. 203–213, 2012.
- [12] J. Sánchez-Oro, A. D. López-Sánchez, and J. M. Colmenar, "A multiobjective parallel variable neighborhood search for the bi-

objective obnoxious p-median problem," *Optimization Letters*, pp. 1–31, 2021.

Copyright © 2023 by the authors. This is an open access article distributed under the Creative Commons Attribution License which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited ([CC BY 4.0](https://creativecommons.org/licenses/by/4.0/)).



**Méziane Aider** is a full professor of operations research at USTHB (Université des Sciences et de la Technologie Houari-Boumediène), Algeria. He is the head of the Laboratory of Operations Research and Mathematics of Decision (LaROMaD). His research work focuses on single and multi-objective combinatorial optimization, graph theory, and didactics of mathematics. He is a regular reviewer for more than twenty international journals.



**Ilham-Aida Azzi** is a PhD student at the USTHB (Université des Sciences et de la Technologie Houari-Boumediène), Algeria. She received her BS in 2011 at the USTHB, and she got her MS in operational research, risk, management and negotiation in 2013 from the same University.



**Mhand Hifi** is a full professor of computer science and operations research at UPJV (Université de Picardie Jules Verne), France. He is the head of the laboratory EPROAD of the UPJV. He is area editor and academic editor for several international journals: CAIE, Advances in OR, AJIBM, etc. His research interest is NP hard combinatorial optimization (sequential and parallel optimization) applied to logistic and OR problems.