New Variable Step-Size Blind Equalization Based on Modified Constant Modulus Algorithm

Roozbeh Hamzehyan, Reza Dianat, and Najmeh Cheraghi Shirazi

***Abstract*—This paper presents a new blind equalization algorithm based on MCMA to attain fast convergence speed and low steady-state error. The channel equalization without resorting to training sequence is called blind equalization. The CMA (Constant Modulus Algorithm) and MCMA (Modified Constant Modulus Algorithm) are two widely referenced algorithms for blind equalization of a QAM system. These algorithms exhibit very slow convergence rates and large steady-state mean square error when compared to algorithms employed in conventional equalization schemes. To obtain better results, we used varying step-size in MCMA, based on estimate of error at the output of equalizer. Simulation results show that the proposed algorithm has a better convergence rates and lower steady state error in comparison to CMA and MCMA algorithms.**

***Index Terms*—Blind equalization, constant modulus algorithm (CMA), inter symbol interference (ISI), modifiled constant modulus algorithm (MCMA)**

I. Introduction

As the applications of digital communication increase, demands for higher data transmission rate and efficient spectrum utilization become more important. However the inter-symbol interference (ISI) caused by multi path and doppler effect spoils transmission performance seriously [1]. To solve the ISI problem, many algorithms and structures have been proposed. The common solution to reduce the effect of ISI is adaptive equalization. Adaptive algorithms are used to update coefficients of equalizer when a channel is unknown and time-varying. Conventionally, the initialization of coefficients is finished by communicating a training sequence from transmitter. Then it is followed by a decision direct mode for normal reception of data. To mitigate the channel distortion problem, many algorithms and structures are suggested.

However, there are some drawbacks for conventional equalizer. This approach will reduce data rate, because we need additional bandwidth to transmit training sequence. Furthermore, for multi-point communication system, it is impractical that the system transmits training sequence to a user while others are waiting. Equalizing a channel without training sequence is known as blind or self-recovering equalization [2]-[7]. Blind equalization does not depend on training sequence to equalize the channel. Blind algorithms use statistical properties of input signal to equalize a channel.

The constant modulus algorithm (CMA) is one of the most widely used blind equalization algorithms [7]. Since the CMA is invariant to a phase rotation in the constellation, the equalizer output signal constellation suffers from an arbitrary phase rotation. A rotator that can rotate the constellation back in the right position at the output of the equalizer is therefore needed in steady-state operation. Oh and Chin [8], and Yang et al. [9] proposed a modified CMA (MCMA) called the multi modulus algorithm (MMA). In this algorithm, joint blind equalization and carrier-phase recovery may be simultaneously accomplished, eliminating the need for a rotator to perform separate constellation-phase recovery in steady-state operation.

II. Channel Base-Band Model

The base-band model of a blind channel equalization system is shown in Fig. 1. Inter-symbol interference, additive noise and carrier frequency offset are considered.

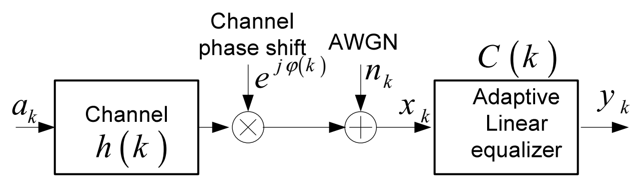

Fig. 1. Base band model of a blind channel equalization system.

Then a received signal x_k can be written as:

$$x_k = \sum_i a_i h_{k-i} e^{j\varphi(k)} + n_k \tag{1}$$

where h_k is the overall complex baseband equivalent impulse response of the transmitter filter, unknown channel and receiver filter. The input data sequence a_k is assumed to be independent, identically distributed (i.i.d.) random variable with zero mean and variance $E[|a_k|^2]$. $\varphi(k)$ is a carrier phase error. n_k is assumed to be additive white Gaussian noise (AWGN) with zero-mean and variance $E[|n_k|^2]=\sigma_n^2$ and to be statistically independent of a_k.

Let $\mathbf{C}(k)$ denote the impulse response of the equalizer in Fig. 1, we get the equalizer output:

$$y_k = \mathbf{X}^T(k)\mathbf{C}(k) \tag{2}$$

where $\mathbf{C}^T(k) = [c_k, c_{k-1}, \ldots, c_{k-N+1}]$ is the equalizer tap

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weights vector, and $\mathbf{X}(k)=[x_k, x_{k-1},...,x_{k-N+1}]$ is the equalizer input data vector. *N* is the length of the equalizer tap weights. In this notation superscript T stands for transpose of vector.

III. Constant Modulus Algoritjm (CMA)

The CMA, proposed by Godard[7], is the most popular technique for blind equalization. It is easy to implement by circuit design. Consider the base-band model of a digital communication channel characterized by finite impulse response (FIR) filter and additive white Gaussian noise. The received signal of equalizer is given in (1). In order to remove effect of channel distortion, we use the equalizer to eliminate this effect. The equalizer output is as in (2). The cost function of CMA is defined as:

$$\mathcal{D}_k^{(2)} = E[(|x_k|^2 - R_2)^2] \quad (3)$$

In this equation R_2 is reference signal depending on statistical properties of input signal x_k. It is defined as:

$$R_2 = \frac{E[|a_k|^4]}{E[|a_k|^2]} \quad (4)$$

Using a stochastic gradient algorithm like LMS, we obtain update equation of CMA:

$$\mathbf{C}(k+1) = \mathbf{C}(k) - \lambda \mathbf{X}^*(k) y_k (|y_k|^2 - R_2) \quad (5)$$

where λ is the step-size parameter and the asterisk denotes complex conjugation. Error signal of CMA is**:**

$$e_k = y_k \left(|y_k|^2 - R_2 \right) \quad (6)$$

For CMA update equation (5), assuming $e_k = 0$ at perfect equalization, we have $|y_k|^2 - R_2 = 0$. It mean CMA attempts to make the equalizer output lie on the circle with radius $\sqrt{R_2}$. Since the cost function is based only on the equalizer output modulus, so CMA is the phase blind algorithm. Furthermore, if the frequency offset exists in equalizer output, the output constellation will spin. Godard demonstrated that the cost function can be applied even to non-constant modulus signals such as rectangular QAM constellations [7].

IV. Modified Constant Modulus Algoritjm (MCMA)

When updating equalizer coefficient by CMA, the equalizer output will have an arbitrary rotation. By modifying the CMA, new algorithm is proposed [8]. MCMA can compensate phase error and slight frequency offset. Modifying cost function of CMA to the form of cost function for real and imaginary parts. The modified cost function is:

$$J_k = J_{k,R} + J_{k,I} \quad (7)$$

where J_R and J_I are the cost functions for real and imaginary parts of equalizer output, $y(k)=Re\{y_k\} + Im\{y_k\}$, respectively, and are defined as:

$$J_{k,R} = E[(|\mathcal{R}e\{y_k\}|^2 - R_{2,R})]^2 \quad (8)$$

$$J_{k,I} = E[(|\mathcal{I}m\{y_k\}|^2 - R_{2,I})]^2 \quad (9)$$

where $R_{2,R}$ and $R_{2,I}$ are reference constant for real and imaginary parts respectively:

$$R_{2,R} = \frac{E(|\mathcal{R}e\{a_k\}|^4)}{E(|\mathcal{R}e\{a_k\}|^2)} \quad (10)$$

$$R_{2,I} = \frac{E(|\mathcal{I}m\{a_k\}|^4)}{E(|\mathcal{I}m\{a_k\}|^2)} \quad (11)$$

Using a stochastic gradient algorithm, blind error of MCMA is obtained as $e_k = \mathcal{R}e\{e_k\} + j\,\mathcal{I}m\{e_k\}$, then:

$$\mathcal{R}e\{e_k\} = \mathcal{R}e\{y_k\}(|\mathcal{R}e\{y_k\}|^2 - R_{2,R}) \quad (12)$$

$$\mathcal{I}m\{e_k\} = \mathcal{I}m\{y_k\}(|\mathcal{I}m\{y_k\}|^2 - R_{2,I}) \quad (13)$$

And the update equation of MCMA is**:**

$$\mathbf{C}(k+1) = \mathbf{C}(k) - \lambda \mathbf{X}^*(k) e_k \quad (14)$$

The update equation of MCMA is similar to that of CMA. For the real channel, the MCMA leads to CMA. From MCMA update equation (14) and assuming $Re\{e_k\} = 0$ and $Im\{e_k\} = 0$ at perfect equalization, we obtain $|\mathcal{R}e\{y_k\}|^2 - R_{2,R} = 0$ and $|\mathcal{I}m\{y_k\}|^2 - R_{2,I} = 0$. It means MCMA attempts to make the real part of equalizer output lie on $\pm\sqrt{R_{2,R}}$, and imaginary part of equalizer output lie on $\pm\sqrt{R_{2,I}}$. With this modification, the MCMA can recover an arbitrary phase rotation due to channel effect and slight frequency offset.

V. Proposed Algorithm

In previous sections we considered two widely references algorithms for blind equalization of QAM system. These algorithms exhibit very slow convergence rates when compared to the algorithms employed in conventional data aided equalization schemes. Usually, because of stability considerations, the values of λ that can be used in the blind equalization algorithms are smaller than the corresponding values used in the LMS algorithm.

Since λ determines the rate of convergence of these types of algorithms, blind equalizers in general exhibit very slow convergence rates. Additionally, consider a data constellation and a blind equalization algorithm for which upon convergence, the error term e_k does not go to zero. In such a situation, for the same steady state mean square error (MSE), the value of λ that is used in the blind equalization algorithm has to be significantly smaller than its permissible value for the LMS algorithm. This is one of the reasons for the slow convergence of blind equalization algorithms.

The proposed algorithm is based on MCMA, since the MCMA can recover an arbitrary phase rotation due to channel effect and slight frequency offset. The propositions to reach high speed convergence in the transient state and low MSE error in steady state is to use the algorithm that can adapt λ regarding the position of the current equalizer output. In fact this algorithm makes use of adaptable-radius decision circles to determine λ that is used for MCMA tap update term to adapt the equalizer taps. Identical decision circles, with adaptable radius R, are centered around each constellation point and if the current equalizer output falls within one of these decision circles, the equalizer updates the taps using small λ and if the symbol falls outside of these decision circles, then the taps are updated with larger step-size term (Fig. 2).

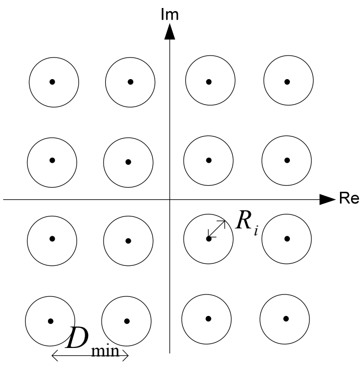

Fig. 2. Decision region for 16QAM

The reason for this approach is that symbols that are close to a given constellation point are assumed to correspond to that constellation point, and hence the resulting hard decision is assumed to be correct and small step-size is used to update equalizer tap weights. Since MCMA with large step-size relies on the statistics of the signal as opposed to hard decisions, it can be used to compute the tap update term when symbols lie farther away from constellation points. The selection of maximum and minimum of this parameter is based on Ungerboeck researches [10].

Another key to proposed algorithm performance is its ability to adapt the radius of the decision circles. This adaptation allows B-CMA to use decision-directed updates more frequently as the channel eye is opened.

Another key to proposed algorithm performance is its ability to adapt the radius of the decision circles. This adaptation allows this algorithm to use large step-size in MCMA updates more frequently as the channel eye is opened. The circle of radius R in Fig. 2 is the decision circle and the value for R is adapted by the algorithm as symbols are processed.

The symbol counter (counter2) is incremented as each symbol is processed. No radius adaptation occurs until at least N symbols have been processed, i.e. until *counter2*= N. At that point, the algorithm computes the number of the previous N symbols that were within neighborhood with distance of R from their corresponding hard decisions.

If more than 90 percent of symbols fall within decision circles, the radius of decision circle and step-size parameter are adapted with $R_{i+1} = \mu_R R_i$ ($0 < \mu_R < 1$) and $\lambda_{i+1} = \mu_\lambda \lambda_i$ ($0 < \mu_\lambda < 1$) respectively. This procedure continues until $R_i = 0.4 D_{\min}$ [8]. After this point this algorithm switches to LMS algorithm. The flowchart of the proposed algorithm is shown in Fig.3.

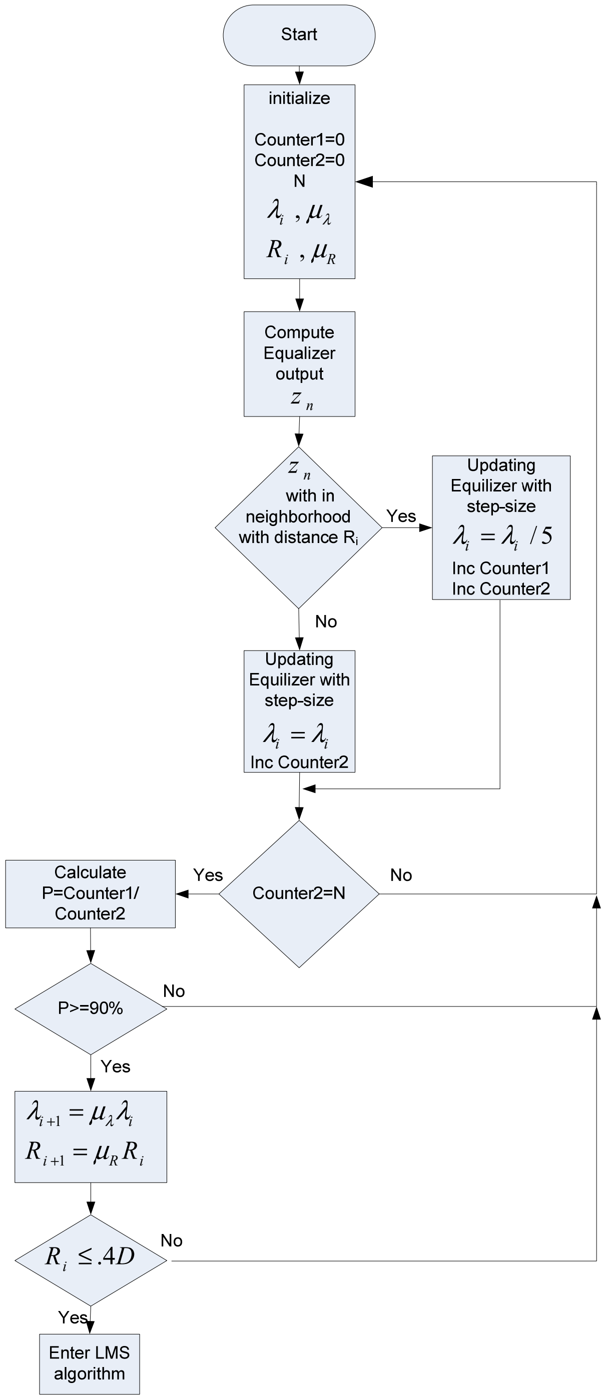

Fig. 3. The flowchart of proposed algorithm

VI. Simulation Results

We have demonstrated the performance of the proposed algorithm, residual ISI and convergence speed, using computer simulations in comparison to CMA and MCMA.

The channel used in simulations was taken from [8]. A 30-tap complex equalizer of transversal filter structure was used and it was initialized so that a center tap was set to one and the other taps were set to zero.

16-QAM constellation was considered in all simulations. The results presented are for the case of white Gaussian noise with an SNR of 30dB at the input of the equalizer. We considered the case of a carrier frequency offset 350 KHz with symbol rate 4.5MS/sec.

First for determination of μ_R , μ_λ and N in the proposed algorithm, we consider one of the parameters constant and change another parameter from 0 to 1 for μ_R , μ_λ and 10 to 500 for *N*. As the measure of performance, we use the residual ISI at the output of the equalizer, which is defined by [8]. As shown in Fig.4 it is necessary to select $\mu_R = 0.4$, $\mu_\lambda = 0.5$ and *N=50* to have maximum convergence speed and minimum residual ISI in equalizer process.

In the second simulation we compared the proposed algorithm with the MCMA and CMA with the scheme for joint equalization and carrier phase recovery. For simulation, the data sequences were generated according to the model of Fig. 1. Fig. 5 shows the constellations after convergence.

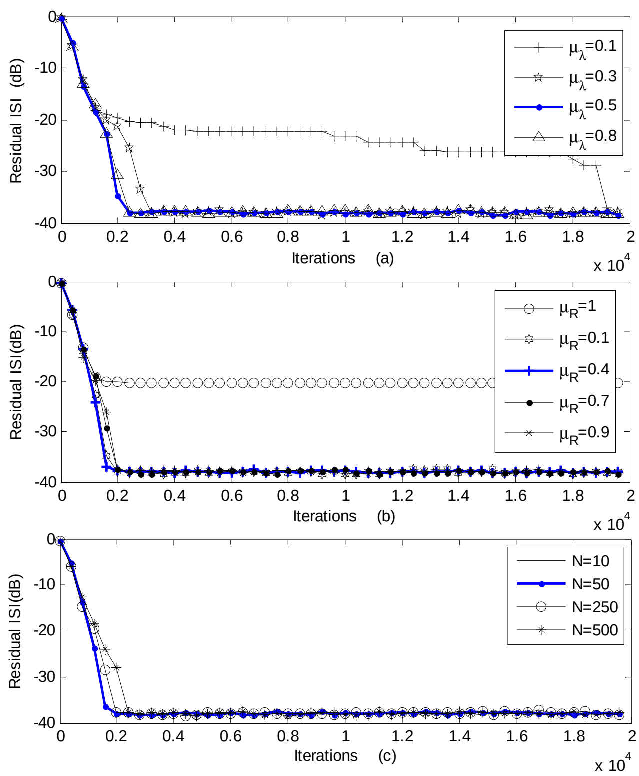

Fig. 4. Residual ISI (a) with constant μ_R (μ_R =0.4) and N (*N=50*) and different amount of μ_λ (b) with constant μ_λ (μ_λ =0.5) and N (*N=50*) and different amount of μ_R (c) with constant μ_λ (μ_λ =0.5) and μ_R (μ_R =0.4)and different amount of *N*.

Fig. 5(b) shows the result for the CMA. It is clear from the orientation of the equalized output constellation that it has an arbitrary phase rotation introduced by the channel, which has not been corrected. Fig. 5(c) and 5(d) show the results for MCMA and for the proposed algorithm respectively. The phase rotation has been recovered for both algorithms. The results show that the proposed algorithm has better convergence than CMA and MCMA.

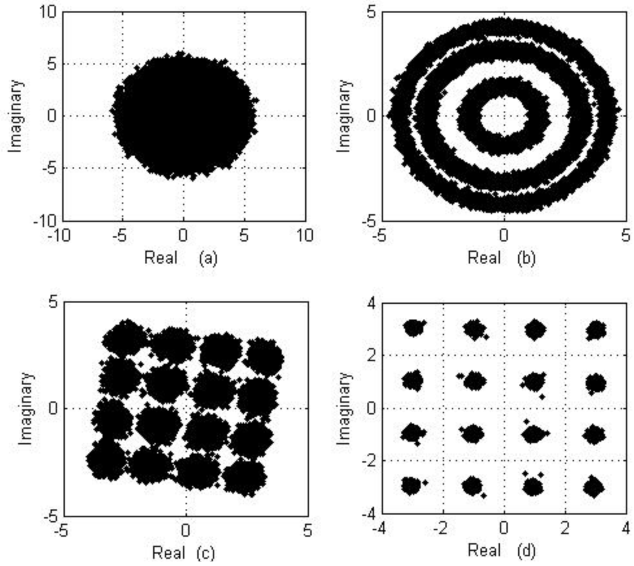

Fig. 5. Constellations of a 16-QAM with a carrier frequency offset. (a) unequalized output, (b) CMA, (c) MCMA, and (d) proposed algorithm.

Next, we compare the convergence behavior and residual mean square error of the CMA and the MCMA and proposed algorithm. Fig. 6 shows the ensemble-averaged mean square error of CMA and MCMA with maximum and minimum amount of step-size.

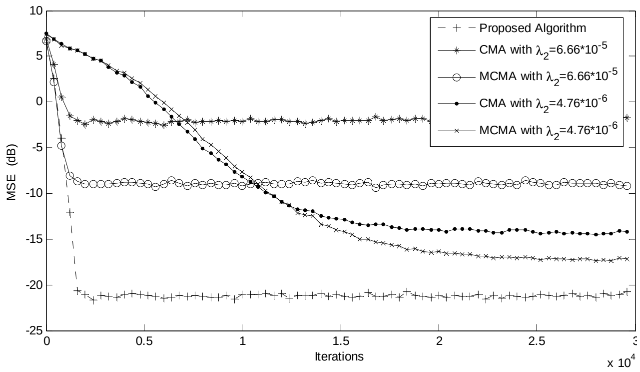

Fig. 6. mean square error in CMA, MCMA and proposed algorithm

The results show that the CMA and MCMA have slower convergence rates in transient state and bigger steady-state mean square error than that of the proposed algorithm. Thus, the new algorithm results in performance enhancement in convergence speed and mean square error than those of the CMA and MCMA.

VII. Conclusion

We proposed a variable step size blind equalization Algorithm based on MCMA that could automatically adapt the step size depending on whether the current equalizer output is in decision circle or not. The effective adaption step size and the radius of decision circle were made continuously variable and decreasing with the decrease in the output error. Such characteristics are beneficial to attain fast convergence speed and low steady-state mean-square error in equalization process. Simulation results demonstrate the effectiveness of the proposed algorithm in comparison with CMA and MCMA.

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