Throughput Maximization Problem in a Cognitive Radio Network

Subhasree Bhattacharjee, Amit Konar and Suman Bhattacharjee

Abstract— **Throughput maximization problem is considered in this paper. Here, throughput analysis is done for fixed allocation of channels. Fixed allocation of channels means number of primary users allocated to a channel is considered to be fixed but the number of users is different for different channels. Primary users allocated to a specific channel compete to access the channel and the cognitive users sense the channel whether the channel is free or not. The transmission probabilities of primary and secondary users are different. In this paper we consider the delay related to channel access of secondary users. We consider number of secondary users is very large. Our objective is to find the optimal number of secondary users to maximize the total throughput of primary and secondary users.**

Index Terms— **primary users, cognitive users, fixed allocation, transmission probability.**

I. INTRODUCTION

Spectral resource becomes scarce today and yet many available spectrum bandwidths are not efficiently used [1], Reports from Federal communication commission (FCC) shows that over 70% of the allocated spectrum is unused in a given time. Cognitive radio (CR) technologies are proposed as a solution [2]. Cognitive radio technology provides us the facility of sharing the licensed radio spectrum between licensed users (primary users) and unlicensed users (secondary users). This is termed as dynamic spectrum sharing in our paper. In the scenario of dynamic spectrum sharing, primary users or licensed users have the privilege of using licensed spectrum band. Secondary users periodically sense the channel whether it is free or not and opportunistically use the spectrum band when primary users do not use it. Thus spectrum utilization improves.

There are many research works regarding dynamic spectrum sharing [3]-[5]. In [6], channel occupancy by the primary users is modeled by Markov chain model. Then a slotted transmission protocol for secondary users using periodic sensing strategy is proposed. Many researchers propose protocols and algorithms to optimize the performance of dynamic spectrum sharing. In [7], the authors model the interaction between primary and secondary users using continuous time Markov chain approach. The proposed primary prioritized Markov approach scheme provides efficient utilization of spectrum among unlicensed users. In [8] throughput trade off problem for a multiple channel cognitive radio network is studied. For maximization of the throughput of cognitive radio network they have designed the optical sensing time and power allocation methods. The effect of cooperation overhead on throughput of secondary network is studied in [9]. Total sensing time and throughput of cognitive network is derived. In [10], throughput and delay for random channel selection scheme base CR system and idle channel selection based CR system is derived. Delay analysis based on success probability of transmission has been studied here. Different MAC protocols are used in CR network for efficient utilization of spectrum. An adaptive MAC protocol for throughput enhancement in cognitive radio network is proposed in [11]. In [12], a decentralized adaptive medium access control protocol with no dedicated global common control channel is proposed. In [13], L. Wang et al. propose the concurrent transmission MAC protocol. This MAC protocol is used to identify the possibility of second link in the presence of first link in an unlicensed spectrum environment. In [14] authors designed a full duplex multi channel MAC protocol for multi-hop transmission in cognitive radio network.

In [15] authors discussed the problem of dynamic sharing of channels between primary and cognitive users with one primary user in each channel. In [16] authors allow variable number of primary users compete for channels. They derive throughput model for primary and secondary users and finally calculate the optimal number of secondary users needed to maximize the throughput. In [17] authors modify the model by calculating discrete convolution to find out the probability mass function of the distribution of channel. They consider packet transmission delay of secondary users when calculating throughput.

The rest of the paper is organized as follows. In section II, the network model is discussed. In section III, throughput analysis for primary and secondary users is done. Section IV deals with results and discussion.

II. NETWORK MODEL

We consider a cognitive radio network with finite number of primary users and infinite number of secondary users. We also consider that if the channel is accessible by secondary users then the users transmit packets with probability q. Primary users transmit with probability p. Let, N and $\widetilde{N}$ are the number of primary users and secondary users respectively. Here, N is finite and $\widetilde{N}$ is infinite. M is the total number of channels. Number of primary users allocated into a channel is fixed but the number of primary users in each channel may be different. Secondary users periodically check the channels and if the channel is idle then secondary users send the packets. When secondary users accessing the channel, at that

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Manuscript received September 8, 2011, revised September 13, 2011.

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time a primary user arrives and then collision occurs between primary and secondary transmissions. Due to collision between these two transmissions secondary users suffer a delay to get access the channels. In this paper we consider this delay of secondary users.

From [16], total throughput for primary users for M channel is

$$T = \sum_{i=1}^{M} X_i . p.(1-p)^{X_i - 1} \qquad (1)$$

Here X_i = number of primary users in the i^th channel,

p = Transmission probability for Primary user,

Throughput of secondary users in a time slot depends on number of accessible channels and number of active secondary users that have data to transmit.

For each channel, the channel will be accessible for secondary user if primary users have no data to transmit. So for ith channel the success probability is $(1-p)^{X_i}$.

Let, Y is a random variable. Yi = 1, if channel is accessible for Secondary user and 0, otherwise.

From [16], p.m.f for Y_i is

$$f_{Y_i}(1) = P_r(Y_i = 1) = (1-p)^{X_i}$$

$$f_{Y_i}(0) = P_r(Y_i = 0) = 1-(1-p)^{X_i}$$

Let, $V = \sum_{i=1}^{M} Y_i$

So, p.m.f. for V is

$$f_V = f_{Y_1} * f_{Y_2} * \dots\dots * f_{Y_M}$$

where * means convolution.

Now, $f_{Y_1} * f_{Y_2}$ involves 3 cases, λ = 0, 1, 2

where λ is a variable. We assume, when

λ =0, no channel is accessible,

λ =1, one channel is accessible,

λ =2, two channels are accessible.

So for λ =0, $f_{Y_1} * f_{Y_2}$

$$= \{1-(1-p)^{X_1}\}\{1-(1-p)^{X_2}\}$$

For λ =1, $f_{Y_1} * f_{Y_2}$

$$= (1-p)^{X_1}\{1-(1-p)^{X_2}\} + (1-p)^{X_2}\{1-(1-p)^{X_1}\}$$

For λ =2, $f_{Y_1} * f_{Y_2}$

$$= (1-p)^{X_1}(1-p)^{X_2}$$

For M channels if λ =M-1

$$f_V = f_{Y_1} * f_{Y_2} * \dots\dots * f_{Y_M}$$

$$= (1-p)^{X_1}.(1-p)^{X_2}.(1-p)^{X_3}\dots\dots(1-p)^{X_{M-1}}.\{1-(1-p)^{X_M}\} +$$

$$(1-p)^{X_1}.(1-p)^{X_2}.(1-p)^{X_3}\dots\dots(1-p)^{X_{M-2}}.\{1-(1-p)^{X_{M-1}}\}.$$

$$(1-p)^{X_M}$$

$$+ (1-p)^{X_1}.(1-p)^{X_2}.(1-p)^{X_3}\dots\dots(1-p)^{X_{M-3}}.\{1-(1-p)^{X_{M-2}}\}$$

$$.(1-p)^{X_{M-1}}.(1-p)^{X_M} + \dots\dots\dots +$$

$$\{1-(1-p)^{X_1}\}.(1-p)^{X_2}.(1-p)^{X_3}\dots\dots(1-p)^{X_M}$$

$$= \sum_{j=1}^{M} (1-p)^{X_1}.(1-p)^{X_2}\dots.(1-p)^{X_{j-1}}.\{1-(1-p)^{X_j}\}.$$

$$(1-p)^{X_{j+1}}\dots\dots(1-p)^{X_M}$$

$$= \sum_{j=1}^{M} (1-p)^{X_1+X_2+\dots+X_{j-1}+X_{j+1}+\dots.+X_M}.\{1-(1-p)^{X_j}\}$$

$$= \sum_{j=1}^{M} (1-p)^{\sum_{i=1}^{M} X_i - X_j}.\{1-(1-p)^{X_j}\}$$

$$f_V = \sum_{j=1}^{M} (1-p)^{N-X_j}.\{1-(1-p)^{X_j}\}$$

$$f_V = f_{Y_1} * f_{Y_2} * \dots\dots * f_{Y_M}$$

$$= \sum_{j=1}^{M} (1-p)^{N-X_j}\{1-(1-p)^{X_j}\} \qquad (2)$$

Now we define a random variable z to denote the number of active secondary users in a time slot. Active secondary users are those whose have data to transmit. We consider z is infinite and follow Poisson distribution. Its p.m.f is

$$f_z(k) = \frac{e^{-m}.m^K}{K!} \qquad (3)$$

where $m = \widetilde{N}q$

Here K is the number of active secondary users and each secondary user has transmission probability q.

Let among M channels, h channels are accessible. So the probability of selecting each channel is 1/h. There should be no collision if it selects an accessible channel that is not selected by any other active secondary user. So the probability of no collision for each channel is $\frac{1}{h}(1-\frac{1}{h})^{K-1}$ [16]. For h channel the probability is $(1-\frac{1}{h})^{K-1}$. For K active Secondary users the throughput can be defined as

$$\tilde{T}(h,k) = K(1-\frac{1}{h})^{K-1} \qquad (4)$$

Let c be the collision probability and Pr_i be the probability that secondary users get access the channel after (i-1)th attempt. That means earlier (i-1) attempts of accessing the channel fail but the ith attempt is successful. So we can write

$$\Pr_i = c^{(i-1)}.(1-c) \qquad (5)$$

We also assume that at most i_{max} times a secondary user tries to get access the channel. Let D_i be the random delay for the ith attempt of accessing the channel and D is the random channel access delay. So average channel access delay

$$D = \sum_{i=1}^{i_{\max}} \Pr_i \sum_{j=1}^{i} D_j \qquad (6)$$

Now from (6) and (7)

$$D = \sum_{i=1}^{i_{\max}} (c^{(i-1)} - c^{i_{\max}}).D_i \qquad (7)$$

Using equation (7), equation (4) becomes,

$$\tilde{T}(h,k) = K(1-\frac{1}{h})^{K-1}.\frac{1}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}$$

$$= K(1-\frac{1}{h})^{(K-1)}.\frac{1}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i} \tag{8}$$

Now from (3) and (8) we have,

$$\tilde{T}(h) = \sum_{k=0}^{\alpha} f_z(k).\tilde{T}(h,k)$$

$$= \sum_{k=0}^{\alpha}\frac{e^{-m}m^k}{k!}.k(1-\frac{1}{h})^{(k-1)}.\frac{1}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}$$

$$= \frac{1}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}.e^{-m}\sum_{k=1}^{\alpha}\frac{m^k}{(k-1)!}.(1-\frac{1}{h})^{(k-1)}$$

$$= \frac{me^{-m}}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}.\sum_{k=1}^{\alpha}\frac{\{m(1-\frac{1}{h})\}^{k-1}}{(k-1)!}...let, k-1=l$$

$$= \frac{me^{-m}}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}.\sum_{l=0}^{\alpha}\frac{\{m(1-\frac{1}{h})\}^{l}}{l!}$$

$$= \frac{me^{-m}}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}.e^{m(1-\frac{1}{h})}$$

$$= \frac{m}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}.e^{-\frac{m}{h}}$$

$$= \frac{\tilde{N}q}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}e^{\frac{-\tilde{N}q}{h}} \tag{9}$$

Now combining (2) and (9) we have,

$$\tilde{T} = \sum_{h=1}^{M} f_v(h)\tilde{T}(h)$$

$$= \sum_{h=1}^{M}[\sum_{j=1}^{M}(1-p)^{N-X_j}\{1-(1-p)^{X_j}\}]$$

$$.\frac{\tilde{N}q}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}e^{\frac{-\tilde{N}q}{h}} \tag{10}$$

From (9) if throughput is maximum, then we have,

$$\frac{q}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i}e^{\frac{-\tilde{N}q}{h}} -$$

$$\frac{q^2\tilde{N}}{h}.\frac{1}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}}).D_i} = 0$$

$$\tilde{N}(h) = \frac{h}{q} \tag{11}$$

$$\tilde{N} = \sum_{h=1}^{M} f_v(h)\tilde{N}(h)$$

$$\tilde{N} = \sum_{h=1}^{M}[\sum_{j=1}^{M}(1-p)^{N-X_j}\{1-(1-p)^{X_j}\}].\frac{h}{q} \tag{12}$$

Now, Total throughput T = Throughput for Secondary user + Throughput for Primary user

$$T=\sum_{h=1}^{M}[\sum_{j=1}^{M}(1-p)^{N-X_j}\{1-(1-p)^{X_j}\}].\frac{\tilde{N}q}{\sum_{i=1}^{i_{\max}}(c^{(i-1)}-c^{i_{\max}})D_i}e^{\frac{-\tilde{N}q}{h}}$$

$$+\sum_{i=1}^{M}X_i.p.(1-p)^{X_i-1}$$

III. RESULTS AND DISCUSSION

In this section the model is simulated to analyze the performance. We draw graph of the total throughput of both primary and secondary users vs. fraction of primary users, i.e. (no of primary users / total number of users). Our aim is to find the optimal number of secondary users for which total throughput is maximized. In Fig. 1, total throughput is plotted with fraction of primary users. Here, number of channel is 10 and number of primary users is 5. The allocation of primary users in 10 channels is assumed as {1,1,1,1,1,0,0,0,0,0}. By varying the number of secondary users we get the plot. Here, three cases are considered regarding the transmission probability of primary users and secondary users. In one case, transmission probability of both primary and secondary users are equal, i.e. (p=q), in second case transmission probability (p) of primary users is greater than the transmission probability (q) of secondary users and in third case reverse is the situation of second case, i.e. p<q. It is observed from the graph that when p<q and fraction of primary users is less than 0.833, total throughput has greater values than the throughput value when p=q and p>q. We vary the number of secondary users from 20 to 0. As the number of secondary users increase, fraction of primary users decreases. When fraction of primary users increases throughput values decrease. Because here primary users' throughput is fixed and as the number of secondary users decrease secondary users' throughput decreases. So, total throughput value decreases.

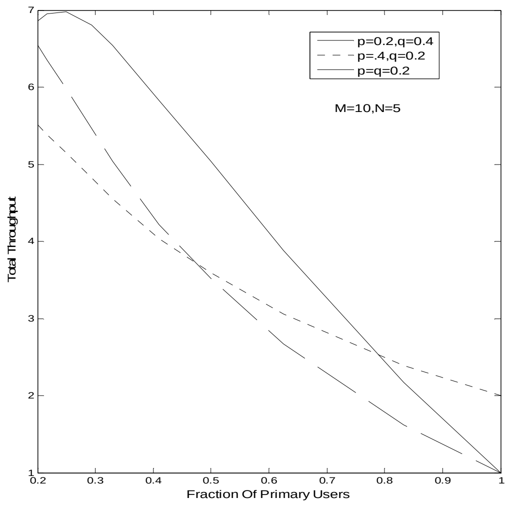

Figure 1. Total Throughput vs. fraction of primary users

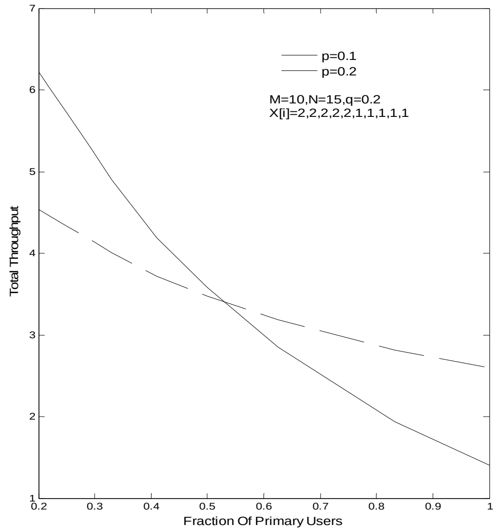

Figure 2.Total Throughput vs. fraction of primary users for different values of p.

Fig. 2 plots total throughput with fraction of primary users keeping transmission probability of secondary user fixed at 0.2. Here, number of channel is 10 and number of primary users is 15. Here, channel allocation is considered as {2,2,2,2,2,1,1,1,1,}. By varying the number of secondary users we get the variation of fraction of primary users. From this graph we see that this graph follows similar trend of results as the previous graph. When the fraction of primary users is greater than 0.5, then the throughput value is greater for lower transmission probability (0.1) of primary users than the higher value (0.2) of transmission probability of primary users.

Fig.3. depicts throughput variation with fraction of primary users for different transmission probabilities of secondary users when transmission probability of primary users is kept equal to 0.2. Fixed allocation of primary users in different channel is considered. The allocation is assumed as {2,2,2,2,2,1,1,1,1,1}.As the transmission probability of secondary users increases the throughput will increase.

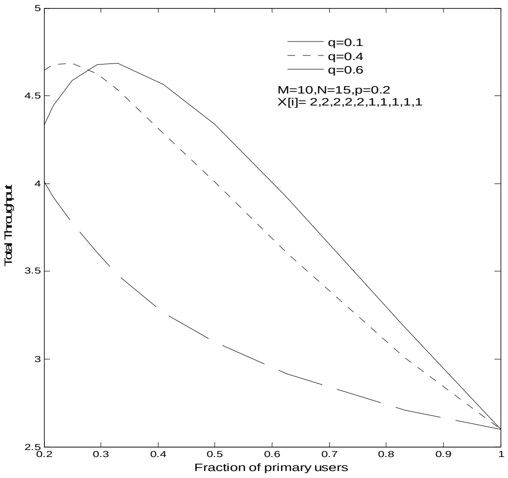

Figure 3.Total Throughput vs. fraction of primary users when p is fixed

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