The Genius of Alan Turing: The Computing Classical Model

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Abstract—**This paper aims to examine the basis of *Calculus* and *computus* from first philosophical principles, having a focus on the internal representations and acts of spontaneity, proper of genius that the concept of creativity is affiliate with. Our guiding author is Alan Turing and we will enquire closely the computing classical model. The paper explores the traditions of computing and philosophy, theorizing about the question of bio-machine hybrids in relation with imagination, the form of representation most free from nature. The first section is called *calculus et computus*. It examines the developments associated with the notions of algorithm, function and rule. In the second section the faculty of imagining is addressed through the abbreviated table, hoping to identify the boundaries both theoretical and practical of the computing classical model, following the seminal paper on computable numbers with Application to the Entscheidungs Problem (1936). We show how much hybridization of ideas fostered by both traditions was to find a place in the imaginary of artificial intelligence. Flanked by intuitions and concepts, imagination, the synthesis of reproduction, is capable of discerning about cosmos through bios and computus, so powerfully as if it sketched ideas in images, as the Turing machine clearly exemplifies.**

Index Terms—**Artificial intelligence, bio-machine hybrids, *calculus et computus*, computing classical model, creativity.**

I. Introduction

The computing classical model has sprung universally and intrinsically from the Turing machine and the universal Turing machine. We aim to trace back the constituent phases of this convolution, by presenting at last a panoptical and functional plane of the *computus* realization, which has, by the passage from incompleteness to effective calculability, endeavored the faculty of imagination and genius towards the critical limits of cosmogenesis.

II. Calculus et Computus

A. From Anthropocentric Humanism to "M-Configurations"

Muhammad Ibn Musa Al-Khwarizmi (850 AD to 780 AD), the Muslim mathematician who first wrote about the system of hindu-arabic numerals and from whose book *Kitab al-Jabr wa al-muqabalah* comes the term "algebra", was also the source of the term "algorithm". The term "function" is a key definition as well.

Manuscript received September 2, 2013; revsied November 4, 2013.

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Kant outlined in a wholly rationalist and empiricist way the importance of the *rule* in the sphere of cognition of human reason, as it metaphorically swerves, like a *curve*, in a very physicist metaphor, between questions called upon to consider due to its nature, but which cannot be answered. This *curve* often drags one specifically *dynamics* metaphor, as this *curve* is, of course, of a special *tension*.

One such example is to be found in *the Critique of Pure Reason* when the author from Königsberg analyses the proposition "all objects are beside each other in space", proceeding to the following:

"'All objects are beside each other in space', is valid only under the limitation that these things are taken as objects of our sensuous intuition. But if I join the condition to the conception and say, 'all things, as external phenomena, are beside each other in space', then the rule is valid universally, and without any limitation [1]."

Kant distinguishes on the one hand, validity under the limitation of objects as part of our sensuous intuition, and on the other hand, universal validity. We shall, henceforth, retain the idea of a curve that slopes, and that it's peculiar *tension* aggregates on the base a universal validity, and validity alone on the top, as it were in another section of the spectrum. In this context the line of *tension* is human reasoning itself. Unlike our daily experience, in this case universality belongs to the base and validity only to the top section of the spectrum. That is why it is associated with *criticism*.

This natural antinomy is not what we consider a *rule*, though. A *rule* is, more justifiably, a sort of natural critical admonition of human reasoning, instructing to experience. In *the Cambridge Companion to Kant and Modern Philosophy* we are alerted to the fact that Kant in the heart of the transcendental deduction (*the Critique of Pure Reason*, Second Edition) considered that even the pure concepts of understanding, such as of mathematics, applied directly to intuition and if one introduces the concept of quantity, it will provide cognition only insofar as there is experience, i.e., empirical intuitions:

"(...) he reminds us of his central theme about empirical knowledge, that the understanding must be "the source of the principles in accordance with which everything (that can even come before us as an object) necessarily stands under rules" [2].

Kant's *Intuitionism* was thus *Empiricist*.

Gödel's *Intuitionism*, on the contrary, was maybe only *empiricist* to the extent of being *realistic* in another way.

However, what is under the spotlight is the *rule* and there is a common denominator in both conceptions, namely what is axiomatically in accordance with the source of principles that constitutes understanding. A *rule* is axiomatically akin to principles and by definition it is of virtually infinite derivability, if proof is not possible.

Remembering that all intuitions are of extensive magnitudes, Kant, who hold a *functionalist* view of the Mind, nevertheless opposed axioms of intuition (Kant, I., *Critique of Pure Reason* [A162/B202]) (connected with the categories of *Unity*, *Plurality* and *Totality*) to axioms of mathematics, as these are by definition synthetic *a priori* and valid according to pure concepts. We could say that accordingly to Kant axioms of mathematics were necessarily related to knowledge (not to experience but intuition only) and axioms of intuition necessarily related to experience (and not necessarily to knowledge).

What Kant acknowledged as axioms of philosophy (Kant, I., *Critique of Pure Reason*, [A733/B761]) was essentially a mechanism of proof, a universal deduction of enlightenment, limited to *Criticism*, and exceptionally *exact* about the referred *curve* and announcement of antinomies.

The *Aufklärung* period heralded *Industrialization* and *Industrialization* did so with *computation*. Yet, in terms of anticipating the established model of computation through Alan Turing's paper *On computable numbers with an Application to the Entscheidungs Problem* (1936) the idea of *calculable by finite means* unfolded a dramatic transformation, profound enough to be called of new *aesthetic perception*, beyond any historical cultural views. Kant's idea on *future metaphysics* was still captive of the medieval conceptual-frame that admitted figures like angels.

Moreover, it was strongly hierarchical and, referring to one of the axioms of intuition, *totally humanist.*

That is to say, if we were to imagine humanity as a domain, according to Kant, there had to be an irrevocable ascent, with knowledge developing as a growing *function* in that domain, with the sum of those arbitrary units of knowledge being the ideal of mankind. This was recognized in different fashion in the contemporaneous era of computation by some philosophers, but in the *totality* of the *concept* could not be appreciated fully in Kant's time abridging the auxiliary *Motus* of machinery, little less with an outer-empowerment of man from alien, unperceptive, automated means. The extension of man by the computing power of modern and forthcoming events was hardly foreseeable even by a genius as the philosopher from Königsberg was.

It's ironic too that such an outstanding *Platonist* and classical ancient world representative as Gödel was, as a mathematician and a logician, to make a major contribution to computation and programming languages, generally by the apagogic method and rebound effect of proof. But there is inevitably a learning process at the birth of every new philosophy and artificial intelligence (Ai) did suffer some setbacks, for example the shift from strong Ai to weak Ai.

This factor, as well as artificial intelligence being considered historically a branch of computer science and not the reverse, serves to underline the huge influence of *Platonism* in western philosophy, which for example forced whitehead to consider European philosophy a footnote of Plato. Prior to Kant's death (1804) in the dawn of XIXth century, we can look back to *computata* and *automaton* history, from *Antikythera* to Frederick ii (the great) of Prussia, whose patronage of arts, science and religious tolerance included the establishment of the Prussian (Berlin) academy of sciences. This supported such proponents of knowledge as Kant himself, but more importantly perhaps french philosophers (French was academy's official language), such as Encyclopedian D'Alembert, Condillac, Maupertuis and the autor of *L'Homme Machine* la Mettrie. Kant's pre-*Industrialism Weltanschauung* was, nevertheless, extremely important epistemologically in respect of the *curve* where *mechanism* joined *machinery*.

Innovative machinery such as the spinning jenny in the wool industry, the cotton gin and jacquard's loom, the water frame and moving factory cogs, the steam engine, and even the discovery of electricity by Volta, were all contemporary to Kant. In respect to Kant, our attention should lie more on the textile machinery than electricity. Kant knew and foresaw the practical implications of the textile machinery engineering power but not quite of electricity, even though Kant was miles away from England, the arena not so much of metaphysical debate, but of machinery debut.

It is the advent of textile machinery and electricity that together provide a first glance of the genesis of *computation*, along the future lines of a Turing machine.

It should not be forgotten that jacquard's loom machinery used punched cards, just the same as early digital XXth century computers.

Babbage and Lovelace together can be considered as the first *in persona modus* of the conceptual pair of hardware and software. This was sort of the *empirical postulate* that history found to give proper rise to *computation* akin to the future ideal Turing machine. Of course a generalized theoretical understanding of magnetism and electricity had yet to be developed. Overall however it seems that Kant was still very *orientated* to the paradigm of the anthropocentric humanist view of XVIIth century of Pascal, Leibniz, Descartes, bacon and Newton, although an advocate of the *criticism* in the new Copernican revolution.

The XVIIth century saw countless developments along the path towards the future concept of *computation*, for instance Leibnizian binary code, machine enterprises such as *Le Pascaline*, the idea of modern age disembodiment of soul and body by Descartes, the scientific strongly inductive method of bacon under the auspice of commanding *nature in action*. Nevertheless, we conclude that the root of these developments was still predominantly *anthropocentric* and *humanist*, and this was the case until shortly post-*industrialization*.

This paper seeks to show that we ought to differentiate two traditions: the *calculus* and the *computus*. Turing's 1936 conception of computability unlocked the future basis for artificial intelligence. The idea of *computare* by means of artificial intelligence, on the lines of Turing's concept, has somehow a distinct imprint, effect and influence than that of *calculus* which was discovered simultaneously by Newton and Leibniz. I believe that this may partly have been due to the fact that Turing was an Englishmen, wandering in Sherbone and Cambridge in difficult times, and lived at the time the British empire, the biggest history has known, was crumbling and of the struggle for his nation's survival in war ii.

There is as much Leibniz in Gödel as there is of jacquard in

Turing. *Calculus* is the representation of man alone just as *computus* is the exemplar of machinery. This seemingly innocent shift or relocation is sufficient, though, to alter consistently culture and civilization, or as Kant would have written, the *metaphysical foundations of natural science*. It is sufficient to alter, in other words, the notion of *rule*.

In order for functions to really *function*, the *calculus* paradigm had to be replaced by that of *computus*.

The progressive move from *anthropocentric humanism*, with so many narcissistic wounds inflicted to man since Galileo and the renaissance, to the loss of *humanism* in Turing ́s time is hard to follow, though we can say that technology is an over-riding concept that continues to become more and more important, matching the move from *calculus* to *computus*. Just as man alienated himself from thinking, in shifting from *calculus* to *computus*, some argue, so some argue that man was alienated from himself. With the passage of time one could say, on the contrary, that history reworks itself. One can cite the subsequent remark by Francis bacon, which emphasizes reasoning over over-confident and sometimes deceptive human memory, bearing in mind Turing´s original idea of ”*m*-configurations”:

“(...) we shall analyze experience and take it to pieces (…) [3].”

B. Turing ́s "Effective Calculable" Humanism

Before closing this first section, we will embrace advances in relation to the notions we postulated at the start, i.e., *function*, *rule*, and *algorithm*. The intrinsic nature of such advances will be compared before we turn more exclusively to Turing in section two of this paper.

What the famous Babylonian clay tablet *YBC 7289* (Fig. 1) indicates to us is a square with diagonals drawn so that a correspondence of numbers from sides to diagonals could be devised, i.e., a coefficient of variables; in the same way, ancient Egyptians knew what we now know by π as the relation of a circumference of a circle to its diameter by means of a *ratio*.

Fig. 1. The Babylonian clay tablet *YBC 7289*.

The Pythagorean theorem was also of such nature that it served as *a priori* proposition that permitted one to see an established truth or *rule* independently, say, of the size or location of the triangles. Another example is the calculation that predicted eclipses of the sun.

There was a widely accepted notion of a *curve* along which different variables could relate to each other by means of an underlying *rule*. Arithmetic only developed into algebra at a later time, so the apparatus of formulae and strings of symbols had not yet encompassed the notion of a *function*. Even before Leibniz coined the term *function* in a rather adventurous prosaic epistolary style, of course there was a general perception of the bridge between one argument to one value considered to be a ‘*function*’.

“In 1694 German mathematician Gottfried Wilhelm Leibniz, codiscoverer of calculus, coined the term *function* (Latin: *Functio*) to mean the slope of the curve, a definition that has very little in common with our current use of the word. The great Swiss mathematician Leonhard Euler (1707–83) recognized the need to make the notion of a relationship between quantities explicit, and he defined the term *function* to mean variable quantity that is dependent upon another quantity. Euler introduced the notation $f(x)$ for “a function of x,” and promoted the idea of a function as a formula. He based all his work in calculus and analysis on this idea, which paved the way for mathematicians to view trigonometric quantities and logarithms as functions. This notion of function subsequently unified many branches of mathematics and physics. (...) advanced texts in mathematics today typically present all three definitions of a function — as a formula, as a set of ordered pairs, and as a mapping — and mathematicians will typically work with all three approaches [4].”

Unhesitatingly we also cite the large discoveries in realm of mathematics, such as that of Oresme, responsible for the first graph (or pictorial function, so to speak) or Napier and Briggs, who worked on tables of logarithms and machinery applications. It ́s really one all-embracing subject, but in my opinion, the first instances of this tendency go back to the renaissance cosmological vision as old as XVth and XVIth centuries, with Cusa and Bruno, and philosophies that were hospitable to the notion of *Omnia Relata Est*, without which the notion of *function* could have not given birth. And so, at this point, we are ready to settle our final conclusions of wide conceptual philosophical relatedness between *function*, *rule* and *algorithm*.

Having set out the concept of function, what it interesting is how the notion of *algorithm* as a list of procedures to approximate and resolve a *function* was crafted into computer science through the design of Turing machines and programming. In this sense, declaratively and procedurally, the *function* had to undergo the passage from a *calculus* paradigm to one of *computus*, so that a *function* could function.

But we shall not forget the slope of the *curve* referred to earlier in this paper. We come to a close by showing how human reason, on common logic grounds, complete to the systematic catalogue of operations it unfolds, by any other means except furnished by experience, is capable of demanding how far it can go, aiming at certitude and clearness, to the matter of critical enquiry of reason. It hesitates in a twofold relation of antinomies, just as similar of that exposed between theoretical and practical cognition. As said, it is a *curve* of a special *tension*, which we can use to correlate *humanism* with *function*.

This is essentially what permitted transcending perspectives about existing philosophical concepts and the bold and daring programme of Ai in computer science in the XXth century. Human bio-machinery was also connected, through a kind of *connectionism*, with the concept of *algorithm*.

Just as critical reason was elaborated by Kant, we verify that Leibniz meant by *function* the *slope* of the curve and the *limit*, in Kantian, terms, therein constructed.

This was to produce a sort of natural deduction table from arithmetic to algebra, conformably proper to the *schema* of numbers to reality. This would be described in descending and ascending perception of quantities, with degrees of continuous generation. Indeed, it is of major importance how the term *rule* migrated to artificial intelligence and belongs nowadays to the jargon of programming languages.

We can now introduce the notion set out by Turing himself in his Princeton Ph.D. Thesis, that of functions being 'effectively calculable', as recalled by Andrew Hodges:

"A function is said to be 'effectively calculable' if its values can be found by some purely mechanical process. Although it is fairly easy to get an intuitive grasp of this idea, it is nevertheless desirable to have some more definite, mathematically expressible definition. Such a definition was first given by Gödel at Princeton in 1934... These functions were described as 'general recursive' by Gödel... Another definition of effective calculability has been given by church... Who identifies it with lambda-definability. (...) We may take this statement literally, understanding by a purely mechanical process one which could be carried out by a machine..." [5]

By this mean Turing could through a *function* approach, fully and clearly articulate how effective calculability in the history of mathematics considered as one *argument*, could, by means of machinery and computation, reach different values, that is, a list of decimals, so that computation and calculability became as one. Turing fused, like jacquard´s or Babbage´s engines interweaved webs of textiles, in intellectual philosophical terms, the tradition of *calculus* with that of new emerging machinery powered by electricity most important above all, to that of *computus.*

Calculus and *computus* are, therefore, all explicitly joined, as are the concepts and dynamics of *function*, *rule* and *algorithm*, through Turing´s all merit and extraordinarily genius.

No wonder too that this ended up as the information era, backed by the growing computation power of Moore´s law, an exponential growth *curve*, passive of various anthropological interpretations.

III. Computus Abbreviated Table

A. Life in the Cell and in the Square

We shall now concentrate on the paper *on computable numbers with application to the Entscheidungs problem* (1936) by Turing. Our discussion will, nevertheless, be linked to other insights. We cannot dismiss the fact that through the idea of *the universal Turing machine* doors were opened in all fields of knowledge, most notably, beyond technicalities in the core of computation, in philosophy of mind and affiliated fields. We have understood how, in the shift from *calculus* to *Computus*, the many-squares or multi-´dimensional' tables of calculus were reduced to a one-dimensional table where squares of symbols finitely run through.

This *reduction* was of a logarithm *exponentiation* kind. The insight about the universal Turing machine is precisely this intangible asset. So too in law and moral philosophy the extraordinary role of constitutions is one balance between *reduction* in things and *exponentiation* of ideas and liberties.

Just as "pairs" were to be one possibility of devising *Functions*, as seen above, so we find in Turing´s paper:

"The possible behaviour of the machine at any moment is determined by the *m*-configuration q and the scanned symbol *s*(*r*). This pair *q*, *s* (*r*) will be called the 'configuration': thus the configuration determines the possible behaviour of the machine." [6]

The so called *universal Turing machine* is only depicted in section *6* of the paper, when Turing aims only to describe *computing machines*. Turing has an interesting view about number theory, as he considers *expressivity* instead of classes or sets. It is the expression of variables in terms of computer numbers that he targets, but this does not mean that they are not concrete *functions* or *real* by principle. This topic is not to be underestimated, as since antiquity we have called *naturals* the class of numbers which are most easily intuited in empirical terms, and now Turing followed a path that is more *Expressivist* than *essentialist* or platonic. His approach provided insightful technical details, of both theoretical and machinery solutions, which bonded *calculus* to decimals, decimals to *computus*, and, thus, in perspective, *calculus* to *computus*.

Decimals are just one way of expressing numbers, a least expensive currency of numbers, so to speak. Turing also discusses the approach of 'predicates', which can be extended outside the realms of traditional number theory, by resorting to understanding exactly how *expressivity* of numbers is just about *predication* inside number theory.

Turing would have agreed with the proposition that numbers are just one limited example of mathematical predicates (they do not directly entail conceptual framework, such as "divisibility", "primality", "ideals", "greatest common Divi- sors" or "unique factorization") but in such neutral limitation, of akin kind of human´s memory limitation, they have the power to resort functions so complex that they surpass its natural limits, equally as man was, in spite of its 'short memory', capable of industrializing primes in tables and, ultimately, capable of resorting different theoretical and practical choices in the very difficult *even* slope of antinomies. That´s why Turing, rather provocatively uses the adverb "naturally" in the expression:

"(...) all numbers which could naturally be regarded as computable [6]."

We must understand that the class of computable numbers is enumerable, but it does not include all definable numbers. Philosophically, that the class of definable numbers is non-congruent (in gauss' terms) with enumerable numbers is very interesting and close to Gödel's legacy. Analyzing computation principles, the reader is confronted with the paradox of infinity. Where can we find non-definable numbers if "number" is basically a pointwise definition, basically restricting a quantity to some unity, at least according to Newton´s classical "definition"? This is similar to berry´s paradox: "the smallest possible integer not definable by a given number of words".

In Gödel´s work, infinity is a thing to marvel at, with so much ineffable relations. Turing is very aware of this, and so he say his results approach Gödel´s. Turing says too that the Hilbertian *Entscheidungs Problem* cannot have a solution. Nevertheless, as this is one important element to follow, we call attention to the comparison: just as one function draws a graph or *curve* and designs a *top* and *bottom* field of values, originally itself the meaning of *function* – the subfield below the *curve* to be more precise – so too with computation we have outlined *above*, in Gödel´s fashion, the unsolvable Hilbert problem, and *below* Alonzo´s "effective calculability" equivalent to Turing´s "computability".

The genius of Turing was to envisage the empowerment out of the vanishing belief in mathematics after the incompleteness theorem let out by Gödel. Turing was to discover the sound basis for incompleteness in the almost complete set of computable numbers, and this reverse approach to the Hilbertian problem is, in fact, one excel demonstration of the creative power of imagination. This new engineering shift and vision led to many problem solving techniques, instead of halting stifling investigation of the *Entscheidungs Problem*.

In Section I. Named "computing machines" Turing without any trace of shame, and at a time in history when many were reluctant to do so because *humanism* was under attack by Faustian belief in technology, adhered to the view that impudently attacked the *humanist* belief most profound out of all, which is *anthropocentrism*. He inflicted another narcissistic wound on man, by affirming the comparison between the human brain and computing machines, even though under the acceptance of computation as one *extension* of humankind.

There is, inarguably, a very strong congruence effect from *bios* to *computus*, dragging the same colonizing effect of *life* in the cell and in the square. This corresponds to an Ai argument thoroughly explored by Dennett, for instance, when reasoning about Darwin´s legacy and contemporary philosophy, even though in Turing's case his mor- phogenesis books are often in the shadows. This is why maybe Turing is thinking about humans too, and not only human intervention as constituent of one external factor in choice-machines (*c*-machines) to the goal of insinuating that the human species is, precisely, a *c*-machine.

Having in mind to make things from what is commonest in nature and with the least waste of energy, Turing reached a path designed desirably so much for *bios* as for *computus*.

"A sequence is said to be computable if it can be computed by a circle-free machine. A number is computable if it differs by an integer from the number computed by a circle- free machine [6]."

By developing a model in this way, both the human mind and machinery, what Turing is saying is that nature, as a field of *calculus* (in a restricted sense) encounters *computus*, so that nature itself can be interpreted as truth tables, better said Turing machines, being the infinite tape time, and nature (in the restricted sense of consciousness, and the synthesis of imagination) being the probabilistic multi-dimension squares.

There is also a powerful philosophical move here towards *symbolism*, as many symbols describe many discrete configuration states, which obscures the break from continuous *calculus* to integers and decimals. In fact, Turing exposes a major defect of the human mind, namely its lack of memory. He both enlarges and restricts in scope both *artefacts*: *human mind* and *machinery*.

We are told in many encyclopedia articles, for example, how Turing machines are in many ways more powerful than state of the art computers, since they are not restricted by any memory storage limitations. There are some variations in the model of a Turing machine, for example where one slides only to the right or ones possessing 5-tuple transitions basic states, instead of the classical 4-tuple transitions: (state, character) → (new state, new character ∨ direction), meaning that it is impossible to see what we find in so many printing robots, which print and move all in one move.

There is also a similarity of consequences comparing the inescapably insufficient computability and the self-inspection method of paradoxes when transposed to the problem of a universal Turing machine, namely with the so called "halting problem": one circle-free Turing machine, if analyzed by any other is prone to as much circularity as the *continuum* problem.

The "halting problem" is itself a paradox, as it happens by default in any circle-free Turing machine, and there is even a certain measure of choice for the machine to make when running, for example, when blank fields are encountered. As Turing stipulated in the paper, this behavior applies to operations on any symbol and also on no symbol. Turing used the example of the sort of palindrome "010101" to 4 "*m*-configurations" under alphabetical notation, in a manner that out of the four operations only two can print out one "0" and one "1" for each complete cycle and repeatedly, allowing only one "right" direction.

The philosophy of language has developed significantly due to reasoning through the work of Turing, independently of paradoxes. Language by its very nature can shelter computational m-configurations and *intuitionist* views. Poetry, for example, has attracted senior Germanophile philosophizing, like Goethe, Wittgenstein and Heidegger, and we cannot say that Gödel is not entitled to be out of this group.

Andrew Hodges gives details about how Turing changed his views on *Abductionist* and *intuitionist* views over time, now with the exception of being reversed. At the start he shared a similar to Gödel´s and post´s view when he wrote *systems of logic based on ordinals* (1939, i.e. The pre-war period), but in the later phase he became pro-engineering and computing-aware, and came up with new insights about definability and uncomputable queries (i.e. The war period). This is illustrated clearly in this passage by Hodge:

"Instead, he decided, the scope of the computable encompassed far more than could be captured by explicit instruction notes, and quite enough to include all that human brains did, however creative or original. Certainly, by the end of the war, he was captivated by the prospect of exploring the scope of the computable on a universal Turing machine; and indeed he called it 'building a brain' when talking of his plans to his electronic engineer assistant. For 1) it was conceived from the outset as a universal machine for which arithmetic would be just one application, and 2) Turing sketched a theory of programming, in which instructions could be manipulated as well as data [7]."

B. Nature as "abbreviated table"

Let us recall that in our intellectual venture, we have sought to analyze, in the shift from the tradition of *calculus* to *computus*, precisely one conceptual reality that could have asked borrowed the term “abbreviated table”. This interpretation contains also the seeds to his later conceptions in the paper *the chemical basis of morphogenesis* (1952), which Harbours a strong *connectionist* perspective, something that was indicted by contemporaneous philosopher of mathematics and Ai, roger Penrose, as expounded by Hodges:

“*The argument from continuity in the nervous system:* the nervous system is certainly not a discrete-state machine. A small error in the information about the size of a nervous impulse impinging on a neuron, may make a large difference to the size of the outgoing impulse. It may be argued that, this being so, one cannot expect to be able to mimic the behaviour of the nervous system with a discrete-state system. (…)

But this brings us to Penrose's central objection, which is not to the discreteness of Turing's machine model of the brain, but to its *computability*. Penrose holds that the function of the brain must have evolved by purely physical processes, but that its behaviour is — in fact must be — uncomputable [8].”

The abbreviated “skeleton tables” as Turing calls them, even though they are not central to his argument, are nevertheless fundamental in the way they introduce firstly the expression “*m*-configuration function” or “*m*-function”. This happens so precisely due to the “abbreviated factor”. Symbols of the machine and *m*-configurations, being the only admissible expressions to be contained therein, are thus exposed so to virtually enable copying, comparing and sequencing symbols of any given form.

We say this so as not to close our investigation without demonstrating the role of *functions* as *rules* in the core of *computus*, demonstrating too that the *curve* of *functions* is a sort of infinite parallel between all domains, so to have made possible the bridge from *calculus* to *computus*. We can see this amongst its uncountable extensions, moreover expound to sound basis, of theoretical worlds so set apart as the *continuum* problem and the nervous system continuity, similar in all ways to the other pairs referred already, most notably other than *Calculus* to *Computus*, the passage from incompleteness to effective calculability. We can comprehend that there is some hidden meaning in the expression *skeleton tables* in relation with *m*-functions so to produce at the end the complete tables for the *m*-configurations, bearing in mind Turing ́s intellectual biography. *“skeleton*” is not just meant to signify a raw and incomplete form or declination table to the aim of producing *m*-configurations.

It conveys the idea of computing as a skeleton table for the *bios*, inasmuch as a skeleton holds a body and is emergently the *in reductio* most raw form of the astonishingly rich surrounding interface of the human body, the unique example amongst all vertebrates, namely having genius.

This is the point where implant technology is supposed to start, to demonstrate computing as it were as a prosthetics of *bios* and man, in Turing ́s words, “naturally”, as if it was being said time prosthetic of space. Antinomies, first-order logic recursively axiomatic incompleteness and the *continuum* problem are just about the high-level problems to which correspond some low-level efficiency, as with effective calculability (the Turing machine idea behind computation) and now, in more utopian and prospective style by *the argument from continuity in the nervous system*, one idea that shaped greatly his later works and Ai as an application field of computer science.

In fact, Turing ́s approach enhances a very specific convergence, when the machine finds the symbol from the *m*-configuration farthest to one side, becoming any altered state depending on the finding of the symbol, as if it would represent, in theory, *animats* – animals and materials – or *hybrots* – hybrids and robots – which are circle-free approximations to both *bios* and *computing*. Curiously, the farthest convergence of *cosmos* with *computing* was not, to my knowledge, one debatable issue in Turing ́s mind, at least to have forced him to write about it consistently.

It seems that the *curve* of this problem, equivalent to the time of history, and equivalent to the discovery of functional analysis throughout history, apart from being intrinsic with human ́s perception of all things relatedness, is essentially related to the close gaping of circle-free Turing machines results. Here we have one paradox, thus: Turing ́s machine (in the limit Turing ́s postulated universal Turing machine), supposedly a circle-free machine, is, conversely, one halting problem in prospect, for the single reason that the circle between *computation* and *cosmos* is lessening more and more, and so is, to our era and following, the circle between *computation* and *bios*, at least accordingly to Turing ́s Ai disciples.

The teleological capacity of man is, thus, related to, in like manner, with antinomies, the *continuum* problem, first-order logic recursively axiomatic incompleteness, as with the halting problem of the universal Turing machine. Turing holds, without a doubt, the *cosmos computing* vision, not just a mere *bios computing* vision. One simple *bioscomputing* vision was supplemented, for instance, by some Enactivist accounts, besides simple connectionism, some sensorimotor theories of perception, or *bios* and *cosmos* semiotics impressions.

Turing transplanted one to the other *representational* views, *action to cognition* and *environmental recognition* views, just as the slope depicted *under* the idea of *function* aggregated different values through a *rule*.

We have speculated now of how much and to which extent *morphogenesis* could bias to *cosmogenesis* and in accordance to Turing ́s insights, but this is not the chief goal for this paper. Finding the concept of *function* inside *computus* with Turing, bringing the computing classical model to encounter its foundations in *calculus*, is one ascending historical *curve* too, and was this article ́s main goal, so to ascertain the faculty of imagining the synthesis behind. The aim was also to make an embryonic interpretation of *abbreviated tables*.

IV. Conclusion

In conclusion, I hope to have explained the place of Turing machines in the history of ideas, exquisitely crafted so as to characterize, in Kantian terms, one *image* and *object* capable of, not only by *analogy*, unifying *acts of recognition* in discrete particulars, but, more extensively, to overpass the idea of one receptacle for foreign representations, and *in continuum* reproduce, as consciousness presented to oneself, by the gift of the faculty of imagining, one thoroughgoing synthetic unity, working for the whole of knowledge. Turing, to whom with so much pleasure we have just celebrated a centenary of life and work, has inspired man to all its imaginable and unimaginable heights.

ACKNOWLEDGMENT

My personal thanks go to the generosity of Mr. Ian Thomson.

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