Tactics for Evacuating from an Affected Area

Qi Wei, Lijuan Wang, and Bo Jiang

***Abstract*—This paper considers the problem faced by a group of evacuees must leave from an affected area as quickly as possible. We seek efficient tactics that achieve a bounded ratio of evacuation time without boundary information to that with. Specially, evacuees can communicate with each other during the evacuation.**

In this paper, we restrict the affected area to a convex region in the plane. We analyze this problem in two scenarios: general plane and plane in grid network. In these two scenarios, we present new efficient tactics and analyze the evacuate ratio of tactics, respectively.

***Index Terms*—Evacuation tactic, ratio, convex region, grid network**

I. Introduction

Motivated by the relations to the well-known searching problem and evacuation problem, much attention has recently been devoted to the problem of how to evacuate from an affected area efficiently when an emergency occurs. We restrict the affected area to a convex region in the plane, but boundary information of the area is unknown to affected people. The goal of the evacuees is to reach the boundary of the area as quickly as possible. Specially, the evacuees can communicate with each other during the evacuation.

In general, the performance of a tactic is measured by a competitive ratio [1] which is defined as follows. Let t denote the evacuation time which is the period from beginning of the evacuation to the moment that all the evacuees successfully evacuate from the affected area. Let t_{opt} denote the shortest evacuation time (in this situation, the evacuees know boundary information of the affected area, so they can reach the boundary of the region with shortest distance). Then the competitive ratio k is defined as follows.

$$k = \frac{t}{t_{opt}}$$

Previous work: The evacuation problem has been extensively studied. Chen *et al.* [2] used methods of system simulation to compare the evacuation efficiency of three different networks, which include the grid network that we also consider in this paper. Lu *et al.* [3] and Shekhar *et al.* [4] studied the shortest path algorithm of evacuation with the consideration of capacity constraints and the increasing number of people in time and space. Berman [5] estimate problem in unknown environment by the performance ratio. Burgard [6] consider the evacuate ratio with the cost of strategy as time spent instead of path lengths.

Xu [7] analyzes the evacuation problem in two scenarios: general plane and plane in grid network. In their strategy, evacuees are divided into several groups, when one group G_i reach the boundary of the affected area, other groups come back to the starting point and walk along the path of G_i until they get out of the region. In the scenario of general plane, the evacuate ratio k is equal to 6 when the divided group number is 3.

Our work: The previous studies mainly focus on details of evacuation such as flow and other constraints to analyze the strategy under complete information without considering the actual situation, in which some information in evacuation may be unavailable. In this paper, we restrict that the evacuees don't know the boundary information of the affected area, but they can communicate with each other during the evacuation.

In the strategy of Xu [7], the evacuation groups except G_i (the first reach the boundary of the affected area) should go back to the starting point, and then walk along the path of G_i. It is not efficient enough. In this paper, we present new efficient tactics and analyze the evacuate ratio of tactics, respectively. In the scenario of general plane, the evacuate ratio k is equal to $2+2\sqrt{3}$ when the divided group number is 3.

The rest of this paper is organized as follows. In Section II, we give some basic definitions relevant to this paper. In Section III, we present new efficient tactic and analyze the evacuate ratio in general plane, respectively. In Section IV, we study the evacuation problems of one group and four groups in grid network, respectively. In Section V, we compare the performance of our tactics in different situations and conclude the paper with a discussion of further research and open questions.

II. Notations and Preliminaries

In this paper, we define a convex polygon as a closed polygonal chain with no interior angle equal and less than 180 degrees [8]. Also an edge of a polygon is defined as a line segment forming a part of the polygonal chain, a vertex of a polygon as a point where two polygon edges meet and the boundary of polygon as a polygonal chain. Let P be a convex polygon and O be the origin. The evacuees starting at O in P don't know the boundary information and their location. The evacuees are divided into several groups to evacuate, and their goal is to leave from P as soon as possible. Let G={

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$G_1, G_{2,...}\ G_n\}$ denote a divided group set of the evacuees.

Successful evacuation of the evacuees requires that all of them have reached the boundary of the affected area. The cost of the tactic is the evacuation time t which is the period from beginning of the evacuation to the moment that all the evacuees successfully evacuate from the affected area. The performance of a tactic is measured by a competitive ratio k which is defined as above. Our objective is to minimize the competitive ratio k.

We now list some properties of the evacuation in this paper:

1) The evacuees starting at O in P don't know the boundary information and their location, but they can share on-time information all the time.
2) The evacuees move at the unit speed during the evacuation. In grid network, the origin O is a node in P.
3) When we consider in grid network, assume that the network consists of several grid units with edge length of 1. Evacuees travel along the edges of network and cannot stay on the edge but the node of the network.

III. Scenario 1: General Plane

In this section, we study the evacuation problem in the situation of general plane. Our tactic is described as follows, see Fig. 1.

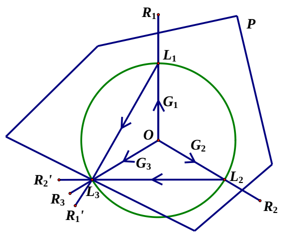

Fig. 1. The tactic EET when n=3.

Equiangular evacuation tactic (EET)

Step 1: Divide the evacuees at O into n (n is odd and $n \geq 3$) groups, let $G=\{G_1, G_{2,...}\ G_n\}$ denote the divided group set of the evacuees.

Step 2: The n groups of evacuees evacuate along n equiangular radials at time t=0, respectively. Let $R=\{R_1, R_{2,...}\ R_n\}$ denote n radials from O in clockwise, where the angle from R_i to R_{i+1} is $\frac{2\pi}{n}$.

Step 3: When one group G_i first reach the boundary of P at time $t=t_i$, other groups that do not arrive at any boundary of P stop and change their evacuation radials. Let L_j denote the location where the group G_j stay at time $t=t_i$. See Fig. 1, the angle between R_i and R_j is $\angle L_jOL_i = |j-i| \times \frac{2\pi}{n}$. If $\angle L_jOL_i < 180°$, $\angle OL_jL_i = \frac{1}{2} \times (180° - \angle L_jOL_i)$, rotate the radial R_j counter-clockwise $180° - \angle OL_jL_i$ about L_j. Otherwise, $\angle L_jOL_i \geqslant 180°$, $\angle OL_jL_i = \frac{1}{2} \times (\angle L_jOL_i - 180°)$, rotate the radial R_j clockwise $180° - \angle OL_jL_i$ about L_j. Let R_j' denote the new radial of group G_j, i.e. radial from L_j to L_i. Then, walk along the new radial R_j' until they get out of P.

In this tactic, the divided group number is odd. Because if the number is even, there must be one group walk along the radial opposite to R_i. In this situation, t is 3 times as t_i, and it is not something we want.

Definition 1 [7]: Let C_e denote a circle with radius r' in P. Let the circle C_f with radius R=maxR' be the largest inner circle of M. If the centre of C_f is O, the circle is called O-center circle of P, denote as C_g with radius r.

Theorem 1: In general plane, the evacuate ratio of EET with n groups is no more than $2+2\sqrt{3}$.

Proof. First, consider the situation in which n=3. See Fig. 1. Suppose that the evacuees are divided into 3 groups(G_1,G_2,G_3), and G_3 first reach the boundary of P. t_{EET} is equal to the time that the last group use in the evacuation. In this situation, $t_{EET}=t_{G1}$. Let d denote the distance which the group walk in the evacuation and d_{opt} denote the shortest distance from O to the boundary of P. Then,

$$k = \frac{t_{EET}}{t_{opt}} = \frac{d_{G_1}}{d_{opt}}.$$

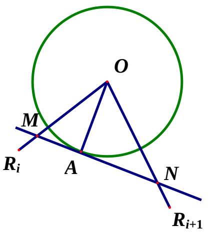

Fig. 2. The relation between d_i and $d_{opt.}$

According to the definition of circle C_g, we know convex region P has at least one intersection point with C_g. Without loss of generality, suppose A is one of the intersection points between R_i and R_{i+1}. See Fig. 2. Tangent line intersects C_g, R_i and R_{i+1} at A, M and N, respectively. Obviously, at time $t=t_3$(when the first group reach the boundary of P), d_i of any group G_i is no more than min$\{OM,ON\}$. Then,

$$d \leq \max\min\{OM, ON\} = \frac{d_{opt}}{\cos\frac{\pi}{n}} \tag{1}$$

$$k = \frac{t_{EET}}{t_{opt}} = \frac{d_{G_1}}{d_{opt}} = \frac{OL_1 + L_1L_3}{d_{opt}} = \frac{OL_1 + 2OL_1\sin\frac{\pi}{3}}{d_{opt}}$$

$$= \frac{OL_3 + 2OL_3\sin\frac{\pi}{3}}{d_{opt}} = \frac{d_3 + 2d_3\sin\frac{\pi}{3}}{d_{opt}}$$

$$\leq \frac{1+2\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = 2+2\sqrt{3} \qquad (from(1))$$

Next consider the situation in which n>3. See Fig. 3. For an example, where n=5.

As discussed above, suppose that the evacuees are divided into n groups (G_1,G_2…G_n), and G_f first reach the boundary of P. t_{EET} is equal to the time that the last group G_q use in the evacuation. In this situation, $t_{EET}=t_{Gq}$. The radial of the last group R_q is the closest one to the opposite radial of R_f (radial of the first group). For example, when n=5, $q= f \pm 2$; when n=n, $q= f \pm \frac{n-1}{2}$.

$$\angle L_q O L_f = \frac{n-1}{2} \times \frac{2\pi}{n} \quad (2)$$

$$k = \frac{t_{EET}}{t_{opt}} = \frac{d_q}{d_{opt}} = \frac{OL_q + L_q L_f}{d_{opt}}$$

$$= \frac{OL_q + 2OL_q \sin\frac{n-1}{2n}\pi}{d_{opt}} \quad (from(2))$$

$$= \frac{OL_f + 2OL_f \sin\frac{n-1}{2n}\pi}{d_{opt}} = \frac{d_f + 2d_f \sin\frac{n-1}{2n}\pi}{d_{opt}}$$

$$\le \frac{1+2\sin\frac{n-1}{2n}\pi}{\cos\frac{\pi}{n}} = \frac{1+2\sin\left(\frac{\pi}{2}-\frac{\pi}{2n}\right)}{\cos\frac{\pi}{n}} \quad (from(1))$$

$$= \frac{1+2\left(\sin\frac{\pi}{2}\cos\frac{\pi}{2n} - \cos\frac{\pi}{2}\sin\frac{\pi}{2n}\right)}{\cos\frac{\pi}{n}}$$

$$= \frac{1+2\cos\frac{\pi}{2n}}{\cos\frac{\pi}{n}} = \frac{1+2\cos\frac{\pi}{2n}}{2\cos^2\frac{\pi}{2n}-1}$$

Let $x=\cos\frac{\pi}{2n}$, n≥3, $x \in \left[\frac{\sqrt{3}}{2}, 1\right)$. Then, $k = \frac{1+2x}{2x^2-1}$.

$\frac{dk}{dx} = \frac{-(2x+1)^2-1}{(2x^2-1)^2} < 0$. So, $k=f(x)$ is a decreasing function,

$$k \le f\left(\frac{\sqrt{3}}{2}\right) = \frac{1+2\times\frac{\sqrt{3}}{2}}{2\times\left(\frac{\sqrt{3}}{2}\right)^2-1} = 2+2\sqrt{3}$$

The proof is thus complete. Theorem 1 follows.

From the discussions made above, the tactic EET becomes more efficient as n increasing. $\lim_{n\to+\infty} k = 3$, the tactic is close to optimal when groups are numerous. So, in this tactic, we can divide the groups of evacuees as many as possible.

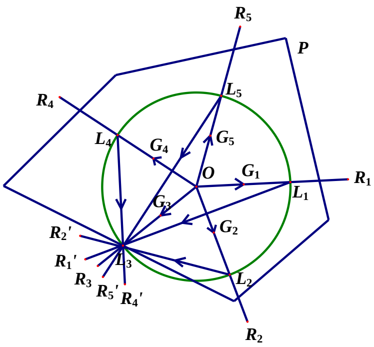

Fig. 3. The tactic EET when n=5.

IV. Scenario 2: Grid Network

Most urban cities have the structure of road network like a grid one [9], See Fig. 4.

In this section, we study the evacuation problem in the situation of grid network. We establish a rectangular coordinate system coincides with grid network with O as the origin. Let $P(x, y)$ denote coordinates of a point in grid network. Let S denote the nearest point to O in the intersections of coordinate axis and P and a denote the path length from S to O.

Fig. 4. Part of the traffic network of Beijing and Jieyang, China.

Definition 2 [7]: For any points $P(x_1, y_1)$ and $P(x_2, y_2)$ in grid network, $L = |x_1 - x_2| + |y_1 - y_2|$ is called evacuation path length.

Theorem 2 [7]: The strategy with boundary information is optimal, and corresponding optimal evacuation time is $t_{opt} = \lceil a \rceil$.

Without loss of authenticity, several situations may occur in the evacuation. Then, we distinguish the following situations.

A. Case 1: One Group

In this situation, all the evacuees leave from the affected area in one group, one path. Our tactic is described as follows, see Fig. 5.

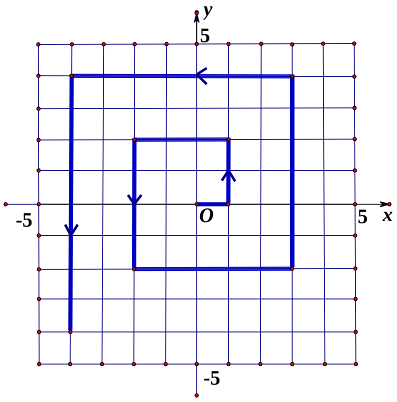

Fig. 5. The tactic OET.

One-group evacuation tactic (OET)

Step 1: when t=0, let i=1, j=1 ($i, j \in N^+$) and G_1 start at point O. Let $D=\{D_1, D_2, D_3, D_4\}=\{+\bar{x}, +\bar{y}, -\bar{x}, -\bar{y}\}$ denote the direction set in grid network.

Step 2: While $j \le 4$, do the followings

1) G_1 walk towards D_j with length of i.
2) j=j+1, i=i+1.
3) if (j > 4) let j=1
4) if (G_1 reach the boundary of P) break.

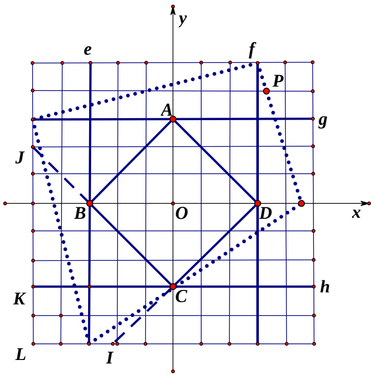

Fig. 6. The proof of lemma 1.

Lemma 1: If the path of tactic OET intersects with the straight line m which passes through point *S* and parallel to the coordinate axis not containing *S*, then reach the coordinate axis containing *S*, it must reach the boundary of *P*.

Proof. Suppose that *S*=*C*, *m*=*h*, as shown in Fig. 6. Make a square *ABCD* which edge expressions are: *X*+*Y*=*a*, *X*-*Y*=*a*, -*X*+*Y*=*a*, -*X*-*Y*=*a*. Obviously, the path length from *O* to any point inside of the closed region *ABCD* $L_{in} \le a$ and the path length from *O* to any point outside of *ABCD* $L_{out} > a$.

There exists no point on the boundary of *P* insides of *ABCD* except *C*. Assume that there exists a point *Q* on the boundary of *P*, which insides of ABCD. Connect *Q* and intersection points of convex region and the axis in which quadrant of point *Q* locates. Thus, $\angle Q > 180°$, a contradiction to the definition of *P*.

The edge e of *P* which contains C and on the left side of *y* axis must in the region JCK or the region KCIL. Otherwise, *P* is not a convex region. If *e* is in the region JCK, when the path of tactic OET intersects with h, it must have reached the boundary of *P*. If *e* is in the region KCIL, when the path of tactic OET reach the axis *y* after intersects with *h*, it must have reached the boundary of *P*.

As discussed above, if S on the other axis *x*+, *y*+, *x*-, i.e. *S*=*A* or *S*=*B* or *S*=*D*, we can also get the same result.

The lemma follows.

Theorem 3: In grid network, the evacuate ratio of OET is no more than $2a+6+\frac{4}{a}$ when *a* is odd; the evacuate ratio of OET is no more than $2a+4+\frac{1}{a}$ when *a* is even.

Proof. Suppose that *a* is odd and *S* is on *y*- axis, as shown in Fig. 7. Let *b* denote the number of edges of the OET path when OET path intersects with m, i.e. the number of edges of the path OH. Let *d*' denote the path length of OET path when it intersects with m and then reach the coordinate axis containing *S*, i.e. the path OI. From lemma1, we know that OI must reach the boundary of *P*.

First, we discuss the relationship between *a* and *b* as well as that between *a* and HI. With the property of OET path, $b=4, HI=2$ when $a=1$; $b=8, HI=4$ when $a=3$. Assume that $b_n=2a_n+2, HI_n=a_n+1$ when $a_n=n$ holds. Because a is odd, $a_{n+1}=n+2, b_{n+1}=b_n+4=2n+6=2a_{n+1}+2, HI_{n+1}=HI_n+2=a_{n+1}+1$. So, $b=2a+2, HI=a+1$ hold.

Then, turn to the evacuate ratio of OET.

$$k=\frac{t_{OET}}{t_{opt}}=\frac{d_{OET}}{d_{opt}}\le\frac{d'}{a}=\frac{OH+HI}{a}$$
$$=\frac{\frac{(1+b)\times b}{2}+a+1}{a}=\frac{2a^2+6a+4}{a}$$
$$=2a+6+\frac{4}{a}$$

Similarly, when *S* is on axis *x*+, *y*+ and *x*-, *k* is equal to $2a-\frac{1}{a}, 2a+2$ and $2a+4+\frac{2}{a}$, respectively. Obviously, $k=2a+6+\frac{4}{a}$ is the biggest one when *S* is on axis *y*-.

As discussed above, suppose that *a* is even, we obtain that $k=2a+4+\frac{1}{a}$ is the biggest one when *S* is on axis *x*+.

The proof is thus complete. Theorem 3 follows.

B. *Case 2: four groups*

In this situation, the evacuees are divided into four groups, our tactic is described as follows, see Fig. 8.

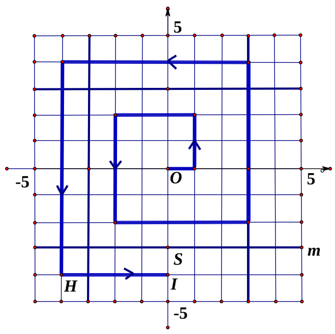

Fig. 7. The proof of theorem 3.

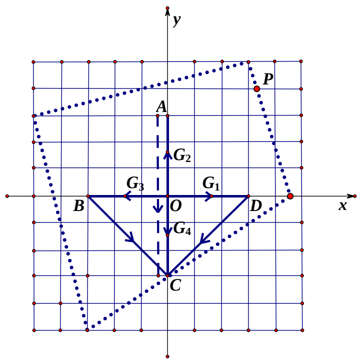

Fig. 8. The tactic FET.

The four-groups evacuation tactic (FET) resembles the tactic EET which we described above. The evacuees are divided into four groups, and walk along the direction of axis *x*+, *y*+, *x*-, *y*- , respectively. When one group G_4 first reach the boundary of *P* at point *C*, other groups turn their directions and go to *C*.

Theorem 4: In grid network, the evacuate ratio of FET is 3.

Proof. As the proof of Theorem 1, we can easily obtain that the evacuate ratio of FET is $k=\frac{3a}{a}=3$.

Theorem 4 follows.

Obviously, in grid network, the tactic FET is optimal when the number of the evacuees $n \geq 4$.

V. Conclusion

In this paper, we study the problem of finding efficient tactics such that the evacuees can leave from the affected area as soon as possible without boundary information. We restrict the affected area to a convex region and analysis this problem in two scenarios. In general plane, we design the tactic EET which evacuate ratio with n groups is $2+2\sqrt{3}$ at most. The tactic EET becomes more efficient as n increasing and it is close to optimal when groups are numerous. See Table I. In grid network, we design tactic OET, FET for evacuation with one group and four groups respectively. The evacuate ratio of OET is no more than $2a+6+\frac{4}{a}$ when a is odd and it is no more than $2a+4+\frac{1}{a}$ when a is even. The evacuate ratio of FET is 3 which is optimal in grid network. See Table II. The performance of FET is best because the number of paths is constrained by structure of network.

TABLE I: The Performance of Tactic EET with Different Group Numbers

Group numbers	n=3	n=5	n=7	n=9
Ratio	$k\approx 5.46$	$k\approx 3.59$	$k\approx 3.27$	$k\approx 3.16$

TABLE II: The Performance of Tactic OET and FET

	a=1	a=2	a=3	a=4
OET	k=12	k=8.5	$k\approx 13.33$	k=12.25
FET	k=3	k=3	k=3	k=3

There are some other related questions which may be interesting for further research. First, suppose that all the evacuees are not located at one point but different points in convex region, how to design an efficient tactic to solve the evacuation problem with this situation. Second, it would be interesting to see how new tactics can be developed so as to efficiently solve the evacuation problem in concave polygon.

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