Nonlinear Optimal Control of Vehicle Active Suspension Considering Actuator Dynamics

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***Abstract*—Considering the nonlinear characteristic of components in vehicle is very important in designing the controller of automobile active suspension system. To show the importance of this effect, a nonlinear optimal control method is employed in this paper. At first, an optimal law is developed for active suspension by the states prediction of a nonlinear quarter car model. The states of quarter car model are first predicted by Taylor series expansion and then a control law is introduced by minimizing the local differences between the predicted and desired states. In this way, the well defined sky hook linear model is selected as the reference model to be tracked by the nonlinear optimal controller. In order to decrease the vertical accelerations and improve the behavior of the reference model, the sky hook model is controlled beforehand by the LQR method. Derived control law has an analytical form which is easy to apply. The simulations show that under the proposed controller, the car has well passenger comfort and safe maneuverability.**

***Index Terms*—Hydraulic dynamics, nonlinear model, nonlinear optimal active suspension control, quarter model, sky hook model.**

I. Introduction

One of the most important problems and noticeable industries of the world automobile manufacture is increase in driving qualification and also more comfort for drivers and passengers. Usual passive suspension systems have mostly concluded of hydraulic dampers and convoluted springs. In order to improvement the comfort of the passengers, passive elements must be located on its soft part. But in order to increase the qualification of driving, the springs and dampers must be selected in the kind of harder ones to eliminate the wheels swings and decrease the pitching and rolling movements. This discrepancy in selection the type of springs and dampers indicates that by a passive suspension system we cannot attain to both two objectives, and we must accept a compromise at this matter. Therefore, the suspension system needs to change the system specifications in a dynamic aspect according to the conditions of the road [1]. Various control strategies such as optimal control [2], nonlinear control [3], robust control [4], adaptive control [5] and intelligent control [6] have been proposed in the past years to control the active suspension system.

But control approaches for active suspensions based on the linear assumption of vehicle model have difficulties in practical application for good performance. In this paper, according to system requirements, an optimal nonlinear approach [7] is applied. The proposed controller has two distinguished features: firstly, it is based on continuous nonlinear model and can handle the model nonlinearity successfully. Secondly, the optimal control law provides the possibility of using lower control energy for achievement of the specified performance and also some physical limits of control input can be satisfied. Application of classic optimal control theory to the nonlinear system requires that the derived nonlinear two point boundary value (TPBV) problem or Hamilton–Jacobi–Bellman (HJB) partial differential equations must solve [8]. It is very difficult or even impossible to find an analytical solution for this problem. Also, numerical computation approaches are not easy to implement and need online dynamic optimization. In this paper, a new optimal predictive approach is utilized to design a nonlinear controller. This method, which employs a nonlinear continuous time dynamic model, leads to an analytical closed form control law which is suitable to implement.

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II. Quarter Model with Hydraulic Actuator

A. Nonlinear Model

The quarter car model shown in Fig. 1 will be used to design active suspension controllers. In Fig. 1 the sprung mass, m_s represents the car chassis, while the unsprung mass, m_{us} , represents the wheel assembly. The parameters k_s and b_s are the suspension stiffness and the damping rate of the suspension that are placed between the car body and the wheel assembly, while the spring, k_t serves to model the compressibility of the pneumatic tire. The variables x_s , x_{us} and r are the car body travel, the wheel travel and the road disturbance, respectively. The force f_s that is applied between the sprung and unsprung masses is generated by means of a hydraulic actuator placed between the two masses. Hence, $f_s = AP_L$ where P_L is the pressure drop across the hydraulic actuator piston and A is the piston area. The hydraulic actuator considered here is a four-way valve-piston system, see [9] for more details. As shown in [10] the rate of change of P_L is given by

$$\frac{V_t}{4\beta_e}\dot{P}_L = Q - C_{tp}P_L - A(\dot{x}_s - \dot{x}_{us}) \tag{1}$$

where V_t is the total actuator volume, β_e is the effective bulk modulus, Q is the load flow and C_{tp} is the total piston leakage coefficient of the piston. In addition, the servo valve load flow equation is given by

$$Q = C_d w x_v \sqrt{\frac{P_s - sign(x_v) P_L}{\rho}} \quad (2)$$

where C_d is the discharge coefficient, w is the spool valve area gradient, x_v is the valve displacement from its ”closed” position, ρ is the hydraulic fluid density and P_s is the supply pressure. However, since we want to include the possibility of the term $P_s - sign(x_v)P_L$ becoming negative, we replace (2) with the corrected flow equation:

$$Q = sign\left[P_s - sign(x_v)P_L\right] C_d w x_v \sqrt{\frac{P_s - sign(x_v) P_L}{\rho}} \quad (3)$$

Finally, the spool valve displacement is controlled by the input to the servo valve u, which could be a current or a voltage. The valve dynamics are approximated by a linear filter with time constant τ .This is a good approximation if the frequency is not too high, and it is regularly used by active suspension designers in industry. The steady-state gain is taken to be one:

$$\dot{x}_v = \frac{1}{\tau}(-x_v + u) \quad (4)$$

where τ is the time constant.

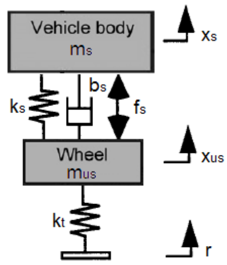


Fig. 1. Quarter car model

In the modeling phase of the control design several specifications are used. The suspension structure is defined by the dynamics of the nonlinear components. The performance demands for ride comfort, road holding and suspension deflection are taken into consideration. The trade-off between performance specifications is defined by a nonlinear function. The force equations of the quarter-car model are

$$\begin{aligned} F_{m_s} &= F_{k_s} + F_{b_s} + f_s \\ F_{m_{us}} &= -\left(F_{k_s} + F_{b_s} + F_{k_t} + f_s\right) \end{aligned} \quad (5)$$

Where the forces from the sprung mass acceleration and the unsprung mass acceleration, the suspension damping force, the suspension spring force, the tire force, respectively, are as follows

$$\begin{aligned} F_{m_s} &= m_s \ddot{x}_s \\ F_{m_{us}} &= m_{us} \ddot{x}_{us} \\ F_{b_s} &= b_s^l(\dot{x}_{us} - \dot{x}_s) + b_s^{nl}\left(\sqrt{|\dot{x}_{us} - \dot{x}_s|}\, sign(\dot{x}_{us} - \dot{x}_s) - |\dot{x}_{us} - \dot{x}_s|\right) \\ F_{k_s} &= k_s^l(x_{us} - x_s) + k_s^{nl}(x_{us} - x_s)^3 \\ F_{k_t} &= k_t(x_{us} - r) \end{aligned} \quad (6)$$

Here, parts of the nonlinear suspension damping b_s are b_s^l, b_s^{nl}. The b_s^l coefficient affects the damping force linearly while b_s^{nl} has a nonlinear impact on the damping characteristics. Parts of the nonlinear suspension stiffness k_s are a linear coefficient k_s^l and a nonlinear one, k_s^{nl}. The state variables are defined to be $x_1 = x_s$, $x_2 = \dot{x}_s$, $x_3 = x_{us}$, $x_4 = \dot{x}_{us}$, $x_5 = \mu P_L$, $x_6 = x_v$. Observe that the pressure drop P_L has been scaled by a constant μ which was taken to be 10^{-6}, the objective of this scaling being to improve numerical conditioning during control design and closed-loop simulation. Then, the nonlinear state-space model for the quarter car dynamics can be written as

$$\dot{X}(t) = f(t, X(t)) + Bu + Dr \quad (7)$$

In this paper, we assume that the suspension deflection limit is $\pm 8cm$ and that the maximum spool valve displacement is $\pm 0.01cm$. The parameter values are taken from reference [10] and are listed in Table I.

TABLE I: PARAMETERS OF THE QUARTER CAR MODEL

$m_s = 290\,kg$	$k_s^l = 16812\,N/M$	$b_s^l = 1000\,Ns/M$
$m_{us} = 59\,kg$	$\gamma = 1.545.10^9\,N/m^{5/2}\,kg^{1/2}$	$b_s^{nl} = 500\,Ns/M$
$\beta = 1s^{-1}$	$\tau = \frac{1}{30}s$	$P_s = 10342500\,pa$
$b_t = 500\,Ns/M$	$k_s^{nl} = 16812\,N/M$	$A = 3.35.10^{-4}\,m^2$

B. Sky Hook Model

In this paper we need a reference model and desired states. The model considered as reference is the skyhook model presented by Karnop for the first time. This model contains a virtual concept and doesn’t exist in reality. In previous works, this Model has been considered as a reference Model, singularly [11]. In Fig. 2, the Sky Hook Model has been shown. Equations of this Model are written in the form of state space as the following equation. Choosing the state variables as in (8) the state space representation of the Sky Hook Model can be written as Equation (9):

$$Y = (r - y_{us}, \dot{y}_{us}, y_{us} - y_s, \dot{y}_s) \quad (8)$$

$$\begin{cases} \dot{y}_1 = \dot{r} - y_2 \\ \dot{y}_2 = \frac{1}{m_{us}}\left[k_t y_1 - k_s y_3 + b_t(\dot{r} - y_2) - u\right] \\ \dot{y}_3 = y_2 - y_4 \\ \dot{y}_4 = \frac{1}{m_s}\left[-b_s y_4 + k_s y_3 + u\right] \end{cases} \quad (9)$$

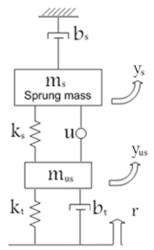


Fig. 2. Sky Hook Used as Reference Model

III. Active Suspension Design

A. Optimal Controller Design for the Sky Hook Model

In this Article, the controlled model of sky Hook is considered for more decrease the vertical acceleration, as reference model, and consequently through definition the desired states based on the controlled model of Sky Hook, non-linear model of automobile suspension, follows the behavior of controlled linear model. The equations in state space for the sky hook can be written as:

$$\dot{Y}(t) = AY(t) + Bu + D\dot{r} \tag{10}$$

Using linear optimal control theory

$$U = -KY \tag{11}$$

B. Nonlinear Optimal Controller Design for Nonlinear Model

The main goal of the control system is to make the actual state x_n ($n = 1,2,...,4$) to follow the desired state x_{nd}. Briefly, the states for the next time interval, $x_n(t+h)$, is first predicted by Taylor series expansion and then the current control u will be found based on continuous minimization of predicted tracking error. Note that h denotes to the predictive period and is a real positive number. Let us first approximate $x_n(t+h)$ by a k th-order Taylor series at t:

$$x_n(t+h) = x_n(t) + h\dot{x}_n(t) + \frac{h^2}{2!}\ddot{x}_n(t) + ... + \frac{h^k}{k!}x_n^{(k)}(t) \tag{12}$$

Now, the key issue is to choose the order k in a way which is suitable for the purposes of controller design on the basis of predictions. The expansion order k is determined as the lowest order of the derivative of state x_n in which the input u first appears explicitly. Hence, state vector $X(t+h)$ is as follows:

$$X(t+h) = L_1\left(t, X(t), W(t), \dot{W}(t), ..., W^{k-1}(t)\right) + L_2\left(t, X(t)\right)u(t) \tag{13}$$

Note that the arguments of functions may be frequently dropped through the rest of paper for simplicity of notations. Now, we consider a performance index that penalizes the next instant tracking error and the current control expenditure in the following form:

$$J[u(t)] = \frac{1}{2}\left(X(t+h) - X_d(t+h)\right)^T Q\left(X(t+h) - X_d(t+h)\right) + \frac{1}{2}Ru^2(t) \tag{14}$$

where the matrix Q is symmetric positive semi-definite, R is symmetric positive definite and X_d is desired states vector. Minimization of the performance index must be sought in order to improve the tracking accuracy of states at the next instant and consequently obtain the optimum behavior of the vehicle. We can expand the desired states in the same manner as we did before:

$$x_{nd}(t+h) = x_{nd}(t) + h\dot{x}_{nd}(t) + \frac{h^2}{2!}\ddot{x}_{nd}(t) + ... + \frac{h^k}{k!}x_{nd}^{(k)}(t) \tag{15}$$

Now, the expanded performance index can be obtained as a function of control input by substituting Eq. (13) into (14) as:

$$J[u(t)] = \frac{1}{2}\left(L_1 + L_2 u - X_d\right)^T Q\left(L_1 + L_2 u - X_d\right) + \frac{1}{2}Ru^2(t) \tag{16}$$

The necessary condition for optimality is

$$\frac{\partial J}{\partial u} = 0 = L_2^T Q\left(L_1 + L_2 u - X_d\right) + Ru \tag{17}$$

which, leads to

$$u = -\left(L_2^T Q L_2 + R\right)^{-1} L_2^T Q\left(L_1 - X_d\right) \tag{18}$$

It is considered that the analytically defined predictive control law Eq. (18), is a closed form which depends on the states of the system and disturbance input. Generally, the proposed control law has two free parameters: the predictive time h and the weighting matrix Q and R. The dynamic performance of the controller is extremely sensitive to the values of these parameters. In the derived control law, the predictive period h is treated as a controller parameter rather than the integration step size. Also, it can be established that a certain degree of robustness in the presence of some modeling uncertainties is achievable through small values of h. We see that the proposed tracking controller technique naturally leads to a special case of feedback linearization. But the current control law (18) has some important advantages over the input/output linearization control. It can be established that the predictive controller is robust in the presence of a class of modeling uncertainties and doesn't need the exact knowledge of the system nonlinearity unlike the feedback linearization. Optimal property of the proposed control law is another important advantage that provides the possibility of limiting the control effort by regulation of weighting matrix Q and R.

IV. Simulation and Discussion

Computer simulations are carried out to verify the effectiveness of the designed nonlinear optimal control system. Matrix Q and R and the predictive period h of the Nonlinear Optimal control are accurately regulate for the computer simulations. Let the set of typical road disturbance be in the form of

$$r(t) = \begin{cases} a\left(1 - \cos(8\pi t)\right) & 0.5 \le t \le 0.75 \\ b & 1 \le t \le 1.25 \\ 0 & otherwise \end{cases} \tag{19}$$

where a and b denote the bump amplitude (Fig. 3). This type of road disturbance has been used by [12] in their studies.

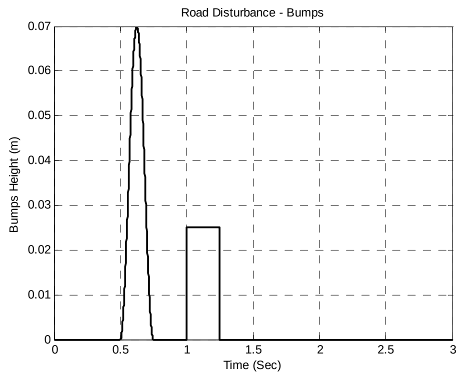
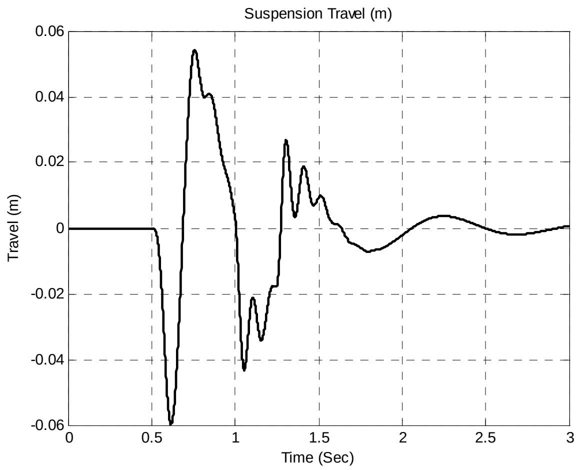


Fig. 3. Typical road disturbance

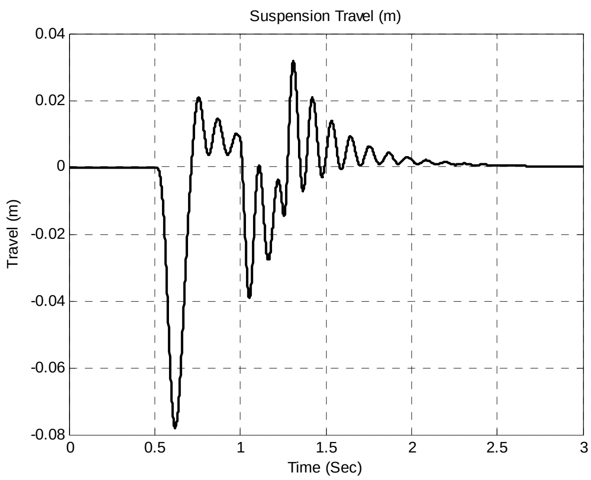
In order to fulfill the objective of designing an active suspension system i.e. to increase the ride comfort and road handling, there are two parameters to be observed in the simulations. The two parameters are the car body acceleration and the wheel deflection. Fig. 4 shows the suspension travel of controller for active suspension system and passive suspension system for comparison purposes. The result shows that the suspension travel within the travel limit i.e. $\pm 8cm$ and the result also shows that the active suspension utilizing the proposed controller, tracked reference model properly. Fig. 5 illustrates clearly how the proposed controller, can effectively absorb the vehicle vibration in comparisons to the passive system. The body acceleration in the proposed control design system is reduced significantly, which guarantee better ride comfort. In this paper, the peak values of the vertical acceleration are also presented. These values indicate the maximum magnitudes of the related acceleration experienced by the vehicle body or passenger. The peak values are calculated as:

$$\|X\|_\infty = \max\left(|x_i|\right) \qquad i = 1,\ldots,n \tag{19}$$

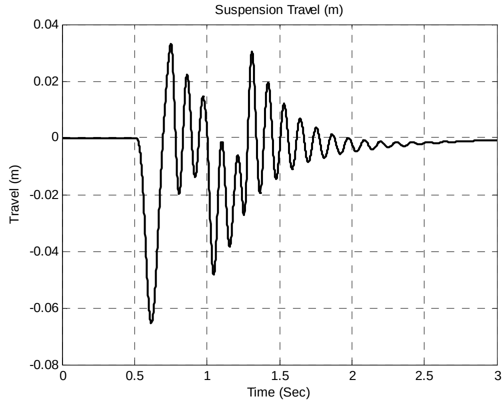
Here, $\|.\|_\infty$ is the ∞-norm. Peak values for the vertical acceleration are depicted in Fig. 6. As seen, the peak values are substantially decreased by the proposed controller.



(a)

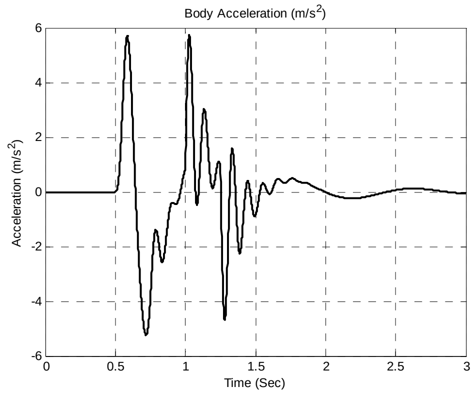


(b)

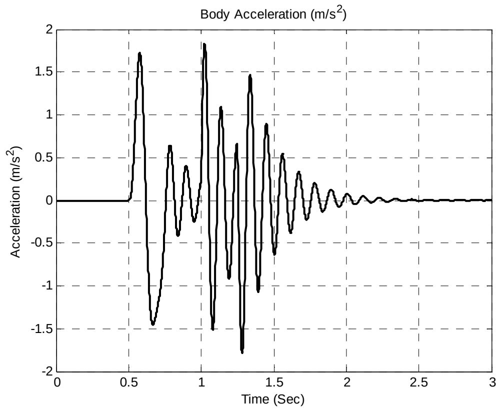


(c)

Fig. 4. The suspension travels: (a) passive suspension, (b) Active suspension by nonlinear optimal controller, (c) Sky Hook Model.



(a)



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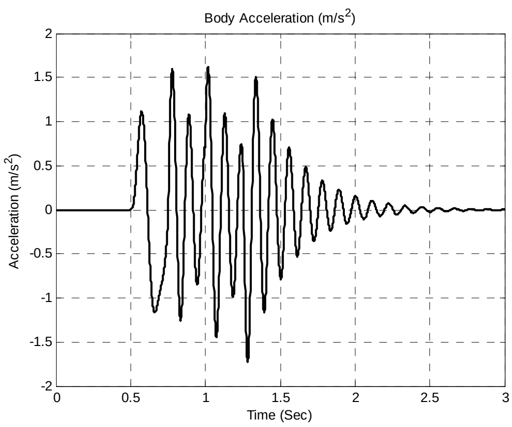

(c)

Fig. 5. The body accelerations: (a) passive suspension, (b) Active suspension by nonlinear optimal controller, (c) Sky Hook Model.

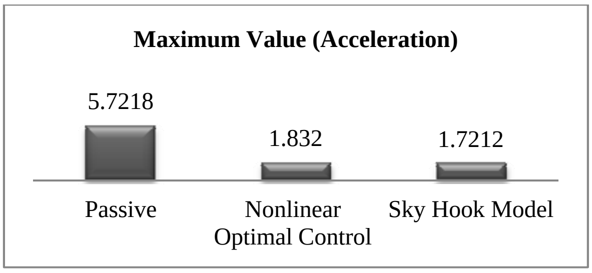

Fig. 6. The peak values for vertical accelerations

V. Conclusion

An automotive suspension supports the vehicle body (sprung mass) on the axles (unstrung mass) and has the following three basic tasks:

1. To isolate a car body from road disturbances in order to provide good ride Quality.

In general, ride quality can be measured by the vertical acceleration of the passenger's locations. Roll and pitch accelerations are not important to improve ride quality even though they are critical quantities for handling issues. Ride quality is very subjective, however, many experimental tests have shown that is closely correlated with the acceleration level and frequency distribution, particularly in the low frequency range (1 - 10 Hz).

2. To keep good road holding and handling on a rough and bumpy road, a winding road, and maneuvers of acceleration lane change and braking.

It is difficult to quantify handling issues because they are subjective. The easiest quantity is road holding which is represented by the tire deflection. Road holding is easier to quantify and is related to the variations in the normal forces. Tire force variations are directly related to tire deflections, reducing tire deflection results in improved traction, braking and cornering.

3. To support the vehicle static weight.

This task is measured by the suspension deflection (rattle space) and depends on the type of suspension used.

According to these aims, an optimization law is developed for suspension system control based on the states prediction of a nonlinear quarter car model. The proposed control law minimized the states tracking errors and led to a special case of feedback linearization. The main features of the proposed controller are as follows.

1) The controller is based on a nonlinear model and can handle the model nonlinearity successfully.
2) The proposed optimal nonlinear control law is given in an analytical form which is easy to implement and the online optimization is not necessary.
3) The simulation results indicate that a stable and safe dynamic behavior through reduced body acceleration can be achieved when the proposed optimal controller is applied.

The proposed control design shows good performances, through time simulation performed on a nonlinear model. Hence this new active strategy exhibits significant improvements on the achieved performances. Moreover, proposed controller compared to passive suspension. The simulations suggest that the proposed controller is an excellent option for active suspension system.

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